Problem Set 6: One-shot approach
FIN 539 Mathematical Finance
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1. State the Fundamental Theorem of Asset Pricing in words.

2. Assume our standard continuous-time model with (1) a single risky asset with constant expected return μ and constant local standard deviation of returns σ, and (2) a riskfree asset with constant risk-free rate r. Recall that the state-price density is $\xi_t = \exp((-r - \kappa^2/2)t - \kappa Z_t)$, where $\kappa = (\mu - r)/\sigma$. Consider the one-shot choice problem for an agent with initial wealth W_0 and consumption only at the horizon T and utility function $u(c_T) = \log(c_T - \bar{c})$ where the constant $\bar{c}$ is the subsistence consumption.

A. Write down the choice problem.

B. Write down the first-order condition and then write down optimal consumption as a function of ξ_T and the Lagrangian multiplier λ.

C. Solve for the wealth process w_t in terms of λ and Z_t. (Recall that for $t > s$, $Z_t - Z_s$ is independent of the history and distributed normally with mean 0 and variance $t - s$. Also, recall that if X is distributed normally with mean a and variance b then $E[\exp(X)] = \exp(a + b/2)$.)

D. Solve for λ, and use this value to restate the wealth process.

E. Solve for the portfolio allocation for the risky asset as a function of wealth and time.