Problem Set 5: Nontraded wealth
FIN 539 Mathematical Finance
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1. What is the difference between priced risk in the Intertemporal CAPM (ICAPM) and the Arbitrage Pricing Theory (APT)?

2. Consider the problem of an agent who has labor income y_t that is not completely spanned by the single risky asset in the economy. We have the choice problem:

Given w_0 and y_0,
choose adapted portfolio θ_t, consumption c_t, and wealth w_t to
maximize $E[\int_{t=0}^{\infty} e^{-\rho t} u(c_t) dt]$ (objective function)
subject to:
$(\forall t)(dw_t = rw_t dt + \theta_t((\mu - r)dt + \sigma dZ_{1t}) - c_t dt + y_t dt)$ (budget constraint),
$(\forall t)(dy_t = ay_t dt + by_t dZ_t)$ (income dynamics),
and
$(\forall t)w_t \geq 0$ (no borrowing).

In this problem, Z_t is 2-dimensional and so is the constant vector b. The first element Z_{1t} of Z_t drives the stock price and can be hedged in the market, while the second element Z_{2t} does not affect stock prices and cannot be hedged in the market. Labor income is affected by both sources of risk. The portfolio θ_t, consumption c_t, wealth w_t, and income y_t are all 1-dimensional. The parameters ρ, r, μ, σ, and a are all constant real numbers.

A. Write down the martingale M_t for this problem.

B. What does M_t represent given the optimal policies for portfolio, consumption, and wealth? What does M_t represent given a suboptimal policy? For $t > s$, what is $E[M_s] - E[M_t]$?

C. Derive the Bellman equation for this problem.

D. Solve for optimal c and θ in terms of derivatives of V.