

Problem Set 5: Nontraded wealth
FIN 539 Mathematical Finance
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Avantika and Jiaen will discuss this homework at the TA session, time TBA.

1. What is the difference between priced risk in the Intertemporal CAPM (ICAPM) and the Arbitrage Pricing Theory (APT)?
2. Consider the problem of an agent who has labor income y_t that is not completely spanned by the single risky asset in the economy. We have the choice problem:

Given w_0 and y_0 ,

choose adapted portfolio θ_t , consumption c_t , and wealth w_t to

maximize $E[\int_{t=0}^{\infty} e^{-\rho t} u(c_t) dt]$ (objective function)

subject to:

$(\forall t)(dw_t = rw_t dt + \theta_t((\mu - r)dt + \sigma dZ_{1t}) - c_t dt + y_t dt)$ (budget constraint),

$(\forall t)(dy_t = ay_t dt + by_t dZ_t)$ (income dynamics),

and

$(\forall t)w_t \geq 0$ (no borrowing).

In this problem, Z_t is 2-dimensional and so is the constant vector b . The first element Z_{1t} of Z_t drives the stock price and can be hedged in the market, while the second element Z_{2t} does not affect stock prices and cannot be hedged in the market. Labor income is affected by both sources of risk. The portfolio θ_t , consumption c_t , wealth w_t , and income y_t are all 1-dimensional. The parameters ρ , r , μ , σ , and a are all constant real numbers.

- A. Write down the martingale M_t for this problem.
- B. What does M_t represent given the optimal policies for portfolio, consumption, and wealth? What does M_t represent given a suboptimal policy? For $t > s$, what is $E[M_s] - E[M_t]$?
- C. Derive the Bellman equation for this problem.
- D. Solve for optimal c and θ in terms of derivatives of V .