Problem Set 1: Probability
FIN 550: Numerical Methods and Optimization in Finance
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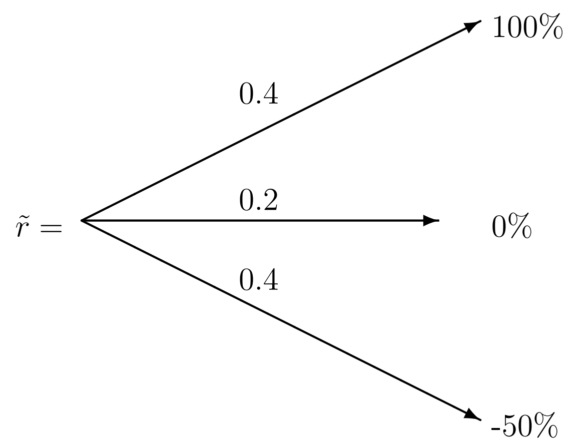
Hand in your answers to Problems 1 and 3 in class on Tuesday, Nov. 6. The answer to Problem 2 will be posted on the web site.

1. Consider a model of stock returns using a trinomial model. The stock return in any period is 100% with probability 0.4, 0% with probability 0.2 and −50% with probability 0.4.



A. Compute the expected return $E[\tilde{r}]$.

B. Compute the variance of return $\text{var}[\tilde{r}]$.

C. Compute the standard deviation of return $\text{std}[\tilde{r}]$

D. Calculate the skewness of the return $\text{skew}[\tilde{r}]$

E. Calculate the kurtosis of the return $\text{kurt}[\tilde{r}]$

2. Assume the stock price S three months from now has an exponential distribution with scale parameter $\theta > 0$, i.e. the density of S is

$$f(S) = \begin{cases} \frac{1}{\theta} e^{-S/\theta} & \text{for } S \geq 0 \\ 0 & \text{for } S < 0 \end{cases}$$

and the cumulative distribution function of S is

$$F(S) = \begin{cases} 1 - e^{-S/\theta} & \text{for } S \geq 0 \\ 0 & \text{for } S < 0 \end{cases}$$

Consider a call option on this stock maturing three months from now with a strike price $X > 0$. The payoff of the call option is

$$C = \max(S - X, 0).$$

A. What is the cumulative distribution function of the option payoff?

B. What is the expected option payoff?

C. What is the variance of the option payoff?

3. Assume the stock price S three months from now has an exponential distribution with scale parameter $\theta > 0$, i.e. the density of S is

$$f(S) = \begin{cases} \frac{1}{\theta} e^{-S/\theta} & \text{for } S \geq 0 \\ 0 & \text{for } S < 0 \end{cases}$$

and the cumulative distribution function of S is

$$F(S) = \begin{cases} 1 - e^{-S/\theta} & \text{for } S \geq 0 \\ 0 & \text{for } S < 0 \end{cases}$$

Consider a sawtooth put option on this stock maturing three months from now with a strike price $X > 0$. The payoff of the sawtooth put option is

$$P = \begin{cases} X - S & S \leq X \\ 2X - S & X < S \leq 2X \\ 0 & 2X < S \end{cases} .$$

A. What is the cumulative distribution function of the option payoff?

B. What is the expected option payoff?

C. What is the variance of the option payoff?

4. (challenger) Let x be distributed uniformly on $[0, 1]$. For each possible realization of x, consider the binary (base 2) representation of x, and let y have the same representation in base 3. For example, if $x = 5/7 = .101101\overline{101}_2$, $y = .101101\overline{101}_3 = 5/13$. (If x is a dyadic rational, the binary representation of x is not unique, but this does not matter because the dyadic rationals are a set of measure 0.) Compute the mean and variance of y.

Note: If you do the challenger, hand in the answer directly to Phil. Challengers are special problems for students of superior preparation or ambition, and are strictly individual efforts.