FIN 550 Practice Exam

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This is a closed-book examination. You may not use texts, notes, a crib sheet, calculator, cell phone, listening device, or any other electronics. Answer all questions as directed on the Blue Books provided. Be sure your answers are clearly marked. There are no trick questions on the exam. Good luck!

0. PLEDGE

The work on this exam is my own alone, and I have conformed with the rules of the exam and the code of the conduct of the Olin School.

Signed name ____

Printed name (write clearly) ____

1. True-False (25 points)

A. Linear programs typically have interior solutions.

B. A local optimum of a convex optimization problem is a global optimum.

C. The Fundamental Theorem of Asset Pricing is about the absence of arbitrage.

D. The product of the eigenvalues equals the trace of a matrix.

F. Slack variables are used to convert inequality constraints into equality constraints.

2. Linear Programming (25 points)

Consider the following linear program:

Choose nonnegative x_1, x_2, and x_3 to
maximize $2x_1 + x_2 + 3x_3$, subject to
$x_1 + 2x_2 + 3x_3 \leq 6$ and
$x_1 + x_2 \leq 3$

A. What is the dual linear program?

B. Is the primal feasible? Is the dual feasible?

C. Infer from the answers in part B: Is the primal bounded? Is the dual bounded?

D. Solve the dual problem.

E. Use the solution to the dual problem to solve the primal problem.

3. REGIME-SWITCHING (25 points) Consider a two-state Markov Chain in continuous time. Regime switches take place at the following rates:

state 1 $\to$ state 2	probability 0.1/year
state 2 $\to$ state 1	probability 0.05/year

Initially (at time $t = 0$), we are in state 2.

a. What is the matrix A in the ODE

$$\pi'(t) = A\pi(t)$$

describing the dynamics of the vector $\pi(t)$ of future regime probabilities?

b. Solve for the eigenvalues of A.

c. Solve for the associated eigenvectors.

d. Write down the general solution of the ODE.

e. Write down the particular solution corresponding to the initial condition that we start in state 2.

f. A project costing \$90,000 has a cash flow of \$6,000/year in state 2 and \$1,000/year in state 1. If the interest rate is 5%/year, does this project have a positive NPV?

4. Kuhn-Tucker Conditions (25 points)

Consider the following optimization problem:

Choose c_u and c_d to
maximize $\frac{1}{2}\log(c_u) + \frac{1}{2}\log(c_d)$, subject to
$\frac{4}{5}(\frac{1}{4}c_u + \frac{3}{4}c_d) \leq 6$.

This is a single-period choice of investment for consumption in a binomial model with log utility, initial wealth of 6, actual probabilities 1/2 and 1/2, risk-neutral probabilities 1/4 and 3/4, and riskfree rate of 25% (and therefore discount factor 4/5).

A. What are the objective function, choice variables, and constraint?

B. What are the Kuhn-Tucker conditions?

C. If we add constraints $c_u \geq 6$ and $c_d \geq 6$, what are the Kuhn-Tucker conditions now?

5. Bonus question (30 bonus points)

A. Solve the optimization problem in problem 4 without the extra constraints in part 4C.

B. Solve the optimization problem in problem 4 with the extra constraints in part 4C.

Useful formulas

For the problem

Choose $x \in \Re^N$ to
maximize $f(x)$
subject to $(\forall i \in \mathcal{E}) g_i(x) = 0$, and
$(\forall i \in \mathcal{I}) g_i(x) \leq 0$,

the Kuhn-Tucker conditions are

$\nabla f(x^*) = \sum_{i \in \mathcal{E} \bigcup \mathcal{I}} \lambda_i \nabla g_i(x^*)$
$(\forall i \in \mathcal{I}) \lambda_i \geq 0$
$\lambda_i g_i(x^*) = 0.$

For the LP

Choose $x \geq 0$ to
minimize $c^\top x$
subject to $Ax \geq b$,

the dual LP is

Choose $y \geq 0$ to
maximize $b^\top y$
subject to $A^\top y \leq c$.