FIN 524B Exam

Phil Dybvig
August 2, 2024

This is a closed-book examination. You may not use texts, notes, a crib sheet, calculator, cell phone, listening device, or any other electronics. Answer all questions as directed on the Blue Books provided. Be sure your answers are clearly marked. There are no trick questions on the exam, but you should read the questions carefully. Good luck!

0. PLEDGE

The work on this exam is my own alone, and I have conformed with the rules of the exam and the code of conduct of the Olin School.

Signed name ____

Printed name (write clearly) ____

1. **True-False** (25 points)

A. Absence of arbitrage is the main concept underlying the binomial option pricing model.

B. When volatility increases, the call price increases and the put price falls.

C. When interest rates are positive, it is always dominated to exercise an American call option on a dividend-paying stock before maturity.

D. It is rare to pay off a mortgage when mortgage interest rates are higher than the existing mortgage's interest rate.

E. A company borrowing money can borrow floating and use an interest rate swap to separate exposure to interest risk from exposure to the firm's credit risk.

2. **Binomial model: futures option** (30 points) Consider the binomial model with $u = 2$, $d = 1/2$, and $r = 5/4$. The stock price is initially \$100 and the stock is not expected to pay any dividends in the next few periods. The actual probabilities are 2/3 for the up state and 1/3 for the down state.

A. What are the risk-neutral probabilities?

B. What is the futures price for a stock futures maturing two periods from now?

C. Price a European put option on the futures with a strike price of \$100 one period from now.

3. **Put-call Parity** (20 points) A stock with a price of \$70 has two listed options with strike equal to \$77 and a maturity a year from now, a put with a price of \$12 and a call with a price of \$10. The one-year risk-free rate is 10% simple interest. Assume interest rates will always be positive.

A. Show that put-call parity does not hold.

B. Suppose it is known that the stock will not pay any dividends during the year. Could the violation of put-call parity be due to the fact that these are American rather than European options? Explain briefly.

4. **Approximate Black-Scholes pricing** Consider a call option on a stock with a local standard deviation of 42%/year. The option has 10 days to maturity (about 1/36 of a year), and is at-the-money in present value, that is, the present value of the strike price equals the stock price. The stock is selling for \$40 and the riskfree rate is 5%/year. No dividend will be paid during the next two weeks.

A. What are the variables T, S, B, and σ to be used in the option formula?

B. What is the call price from the approximation formula?

C. What is the corresponding European put price?

5. Bonus problem (20 bonus points) A stock has price \$80, the interest rate is 10% (simple interest), a call option maturing a year from now with exercise price \$22 has price \$59, and a put option maturing a year from now with exercise price \$33 has price \$2.20. Assume that the options are both American options, and that it is known that the stock will not pay any dividends this year. Show there is arbitrage by describing the strategy to follow.

Some Useful Formulas

Binomial model: if the stock has up factor u and down factor d, and one plus the riskfree rate is r, then the risk-neutral probabilities are:

$$\pi_u^* = \frac{r-d}{u-d}$$

$$\pi_d^* = \frac{u-r}{u-d}$$

and the one-period valuation is

$$\mathrm{price} = \frac{1}{\mathrm{r}}(\pi_\mathrm{u}^* \mathrm{V_u} + \pi_\mathrm{d}^* \mathrm{V_d}).$$

The Black-Scholes call price is

$$C(S,T) = SN(x_1) - BN(x_2),$$

where S is the stock price, $N(\cdot)$ is the cumulative normal distribution function, T is time-to-maturity, $B = Xe^{-rT}$ is the bond price, r is the continuously-compounded riskfree rate, σ is the standard deviation of stock returns (so σ^2 is the local variance),

$$x_1 = \frac{\log(S/B)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T},$$

and

$$x_2 = \frac{\log(S/B)}{\sigma\sqrt{T}} - \frac{1}{2}\sigma\sqrt{T},$$

Note that $\log(\cdot)$ is the natural logarithm.

The Black-Scholes call price can be approximated by

$$\frac{S-B}{2} + 0.4\frac{S+B}{2}\sigma\sqrt{T}.$$

The put-call parity formula is

$$B + C = S + P.$$