Fin 524 final exam

Phil Dybvig
October 19, 2004

This is a closed-book examination. Answer all questions as directed. Mark your answers directly on the examination. On the valuation questions, make sure your answer is clearly indicated. There are no trick questions on the exam. Good luck!

A. Binomial Option Pricing in One Period *40 points*

Riskless bond (interest rate is 10%):

$$100 \longrightarrow 110$$

Stock:

$$120 \begin{array}{l} \nearrow 180 \\ \searrow 108 \end{array}$$

European call with a strike price of 114:

$$? \begin{array}{l} \nearrow ? \\ \searrow ? \end{array}$$

1. What are the payoffs of the European call in the up and down states?

$$? \begin{array}{l} \nearrow 66 \\ \searrow 0 \end{array}$$

2. What are the risk-neutral probabilities for the two states tomorrow?

$$\begin{aligned} r &= 1 + 10\% = 1.1 \\ u &= 180/120 = 1.5 \\ d &= 108/120 = 0.9 \end{aligned}$$

$$\begin{aligned} \pi_u &= \frac{r-d}{u-d} = \frac{1.1-0.9}{1.5-0.9} = \frac{1}{3} \\ \pi_d &= 1 - \pi_u = 1 - \frac{1}{3} = \frac{2}{3} \end{aligned}$$

3. What is the price of the European call today?

$$\frac{1}{1.1}\left(\frac{1}{3}66 + \frac{2}{3}0\right) = 20$$

4. Would the value of the corresponding American call be the same?

Yes. Exercising the option immediately is worth $120 - 114 = 6$ which is less than waiting to maturity which is worth 20.

B. Concepts (multiple choice) *20 points*

1. Which of the following is *not* a common type of option?

a. Call
b. Put
c. Digital
$\boxed{\text{d. Elephant}}$

2. In risk-neutral probabilities, what is the mean percentage change in a stock price?

- $\boxed{\text{a. Riskfree rate}}$
- b. Risk premium
- c. Riskfree rate plus a risk premium
- d. Zero

3. In actual probabilities, what is the mean percentage change in a futures price?

- a. Riskfree rate
- $\boxed{\text{b. Risk premium}}$
- c. Riskfree rate plus a risk premium
- d. Zero

4. A portfolio manager follows a dynamic trading strategy to replicate an equity position plus a put. This is best described by which term?

- a. asset-liability management
- b. portable alpha
- $\boxed{\text{c. portfolio insurance}}$
- d. long-short hedge fund

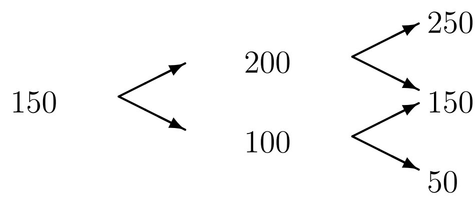
5. Suppose we expect the price of ABC stock to be very volatile but we don't have a view on the direction it will move. Which of the following is the best strategy?

- a. Buy one share and one put
- b. Buy one call and sell one put
- $\boxed{\textbf{c. Buy one put and one call}}$
- d. Sell two calls and buy one bond

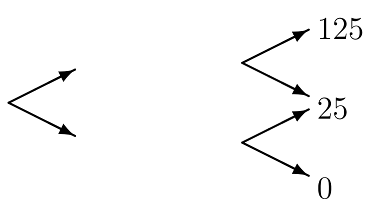
C. Binomial futures option pricing *40 points*

The short riskless interest rate is 25% per period. The two-period corn futures price is \$150 today and will go up or down by \$50 each period, with risk-neutral probabilities 1/2 and 1/2. Consider a European futures call option with an exercise price of \$125 maturing two periods from now.

1. What are futures prices in the tree?



2. What are the call payoffs at maturity?



3. What is the price of the futures call option at each node in the tree?

$$
\begin{tikzcd}[row sep=small]
 & & 125 \\
 & 60 \arrow[ur] \arrow[dr] & \\
28 \arrow[ur] \arrow[dr] & & 25 \\
 & 10 \arrow[ur] \arrow[dr] & \\
 & & 0
\end{tikzcd}
$$

for example,

$$\frac{1}{1.25}\left(\frac{1}{2}125+\frac{1}{2}25\right)=60$$

4. (conceptual question) Why aren't the risk-neutral probabilities 1/2 and 1/2 the same as what would be given by the familiar formulas $\pi_u=(r-d)/(u-d)$ and $\pi_d=(u-r)/(u-d)$ where u and d are computed from the futures price tree?

The familiar formulas assume the risk-neutral expected change in the stock price equal to the riskfree rate, not the risk-neutral expected change in the futures price, which is zero.

D. Bonus question (short answer) *20 points*

In the Black-Scholes model, the call option price is always positive before maturity if the volatility is positive. However, in the binomial model the call option price can be zero before maturity even if the volatility is positive. Why are the two models different?

Since the stock price can only take on a few values in the binomial model, it is possible for the strike to be bigger than all, which means the call is worthless because it is always out of the money. However, the stock price can take on any positive value in Black-Scholes.

Useful Formulas

Binomial model: if the stock has up and down factors u and d and one plus the riskfree rate is r, then the risk-neutral probabilities are

$$\pi_u = \frac{r-d}{u-d}$$
$$\pi_d = \frac{u-r}{u-d}$$

and the one-period option valuation is

$$\text{price} = \frac{1}{r}\left(\pi_u V_u + \pi_d V_d\right).$$

The Black-Scholes call price is

$$C(S,T) = SN(x_1) - BN(x_2),$$

where S is the stock price, $N(\cdot)$ is the cumulative normal distribution function, T is time-to-maturity, B is the bond price $Xe^{-r_f T}$, r_f is the continuously-compounded riskfree rate, σ is the standard deviation of stock returns,

$$x_1 = \frac{\log(S/B)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T},$$

and

$$x_2 = \frac{\log(S/B)}{\sigma\sqrt{T}} - \frac{1}{2}\sigma\sqrt{T}.$$

Note that $\log(\cdot)$ is the natural logorithm.
The Black-Scholes call price can be approximated by

$$\frac{S-B}{2} + .4\frac{S+B}{2}\sigma\sqrt{T}.$$

The put-call parity formula is

$$B + C = S + P.$$