Practice problems for Lecture 2: answers

1. (short answer) Answer each question in no more than one sentence of normal length.

a. Define *American call option*.

An American call option gives the purchaser the right, but not the obligation, to buy one share of stock at a prespecified price at or before the specified maturity date.

b. Buying a call option is buying or selling vol?

buying

c. Buying a put option is a long or short position in the stock?

short

d. Does an European put option price increase or decrease in maturity?

ambiguous

(If it is far enough out of the money it increases because increasing maturity increases the probability the option will move into the money, but if it is far enough in the money it decreases because the strike price you will collect is discounted more when you increase the maturity. An American option price can only increase with maturity because you are allowed to exercise early.)

2. A Simple Option Pricing Problem in One Period

Riskless bond (interest rate is 50%):

$$100 \longrightarrow 150$$

Stock:

$$\begin{tikzcd}[row sep=tiny] & 125 \\ 50 \arrow[ur] \arrow[dr] & \\ & 50 \end{tikzcd}$$

Derivative security (call option with a strike of 80):

$$\begin{tikzcd}[row sep=tiny] & 45 \\ ? \arrow[ur] \arrow[dr] & \\ & 0 \end{tikzcd}$$

a. What is the portfolio of the stock and the bond that replicates the option?

Suppose the replicating portfolio has α_S dollars worth of stock and α_B dollars worth of bonds. Replicating in both states implies that

$$(125/50)\alpha_S + (150/100)\alpha_B = 45 \quad \text{(up state)}$$

and

$$(50/50)\alpha_S + (150/100)\alpha_B = 0 \quad \text{(down state).}$$

Solving for α_S and α_B, we have that $1.5\alpha_S = 45$ or $\alpha_S = 30$ (which is purchase of 3/5 of a share) and $30 + 1.5\alpha_B = 0$ or $\alpha_B = -20$ (which is shorting 1/5 of a bond).

b. What is the option price (given by the price of the replicating portfolio)?

The price of the option is $\alpha_S + \alpha_B = 30 - 20 = 10$.

c. What are the risk-neutral probabilities of the two states? Verify that your answer gives the correct price for the stock, the bond, and the option. Reminder: the formula for the risk-neutral probability of an up move is $(r - d)/(u - d)$.

$$r = \frac{150}{100} = 1.5; u = \frac{125}{50} = 2.5; d = \frac{50}{50} = 1$$

The risk-neutral probability of the up state is $(r-d)/(u-d) = (1.5-1)/(2.5-1) = 1/3$. The risk-neutral probability of the down state is $1 - 1/3 = 2/3$. For the stock, the price is 50 as is given using the risk-neutral pricing:

$$\begin{aligned}
\text{Stock price} &= \frac{1}{1.5}\left(\frac{1}{3}125 + \frac{2}{3}50\right) \\
&= \frac{2}{3}\left(\frac{125}{3} + \frac{100}{3}\right) \\
&= 50.
\end{aligned}$$

Similarly, the bond is correctly priced at 100:

$$\begin{aligned}
\text{Bond price} &= \frac{1}{1.5}\left(\frac{1}{3}150 + \frac{2}{3}150\right) \\
&= \frac{2}{3}150 \\
&= 100.
\end{aligned}$$

And, the option is correctly priced as 10:

$$\begin{aligned}
\text{Call Option price} &= \frac{1}{1.5}\left(\frac{1}{3}45 + \frac{2}{3}0\right) \\
&= \frac{2}{3}15 \\
&= 10.
\end{aligned}$$

d. (thought question—answer in a sentence or two of ordinary length) Suppose we have inherited many shares of the stock and would like to trade them to diversify but we are precluded from trading the stock due to terms of an inheritence. If the inheritence does not preclude trading the option, how can this help us? (Note: do not perform any calculations in this answer.)

If we short a portfolio of options and bonds that replicates the stock, we can undo the risk exposure without violating at least the letter of the terms of our inheritence. (Warning: this is not tax advice! You would need to check the tax law before doing this. Selling a call might be viewed as a constructive sale of the stock, generating unwanted realization of gains.)

3. Two-period model: futures option versus stock option maturing in one period.

Riskless bond (riskfree rate is 25%):

$$16 \longrightarrow 20 \longrightarrow 25$$

Stock price (no dividends):

$$\begin{array}{ccccc} & & & \nearrow & 225 \\ & \nearrow & 120 & \searrow & \\ 64 & & & & 75 \\ & \searrow & 40 & \nearrow & \\ & & & \searrow & 25 \end{array}$$

The actual probabilities are 2/3 for the up state and 1/3 for the down state.

a. For all nodes in the tree, compute the futures price for a futures on the stock, maturing at the end.

The risk-neutral up probability is $(r-d)/(u-d) = (5/4-5/8)/(15/8-5/4) = 1/2$, and the down probability is $1 - 1/2 = 1/2$. The futures is worth the same as the stock at the end. Since the futures is not an investment, we do not discount when we compute the price earlier in the tree.

$$\begin{array}{ccccc} & & & \nearrow & 225 \\ & \nearrow & 150 & \searrow & \\ 100 & & & & 75 \\ & \searrow & 50 & \nearrow & \\ & & & \searrow & 25 \end{array}$$

b. Compute the price of a futures put option, with the put maturing in the middle time, with a strike price of 70, on the futures maturing at the end.

$$\begin{array}{ccl} & \nearrow & 0 \\ ?? & & \\ & \searrow & 70 - 50 = 20 \end{array}$$

The put is an investment, so we do discount in computing its value, which is $\frac{1}{1.25}(\frac{1}{2}0+\frac{1}{2}20)=8$.

c. Compute the price of a put option on the stock, with the put maturing in the middle time with a strike price equal to 70.

$$\begin{tikzcd}[row sep=small] & 0 \\ ?? \arrow[ur] \arrow[dr] & \\ & 70-40=30 \end{tikzcd}$$

This put is also an investment, so the value is $\frac{1}{1.25}(\frac{1}{2}0+\frac{1}{2}30)=12$.