Practice problems for Lecture 1: answers

1. (short answer) Answer each question in no more than one sentence of normal length.

a. Define *arbitrage*.

Arbitrage is a strategy that makes money sometimes and never loses money.

b. Counterparty risk is usually less for futures contracts than for forwards. Why?

The counterparty for traders of a futures contract is usually the exchange, which is backed by the exchange owners. The exchange has little counterparty risk because of daily settlement (limiting exposure to one day of risk) and margin erquirements (with margin accounts available to cover a generous estimate of one day's losses).

c. Name a type of derivative security whose value depends on private attributes as well as market variables.

(two of many possible answers) mortgage-backed security or option to renew a lease

d. Name one consideration in the decision of whether or not to hedge a risk.

(two of many possible answers) If you think you have information better than the market about the likely direction of the price move, you may want a position that would profit from the likely price move, not a hedged position. You may not want to hedge if doing so would introduce an unreasonable amount of variation in accounting numbers.?

2. (convergence trade) Following hurricane damage to the orange groves in Florida, there is a significant difference between the spot price 135¢/pound of frozen concentrated orange juice and the six-month futures price of 160¢/pound. Grade A frozen concentrated orange juice contracts FCOJ-A are listed on ICE. Each contract is for 15,000 pounds of frozen concentrated orange juice. (See https://www.theice.com/products/30/specs for more detail on these

contracts.) One contract worth of frozen concentrated orange juice can be stored at a qualified warehouse for \$600/month. Delivery of an initial purchase or delivery on a futures contract is made in physical orange juice at one of the official warehouses.

a. Supposing you can borrow money at 5% straight interest, is there an arb from taking advantage of the expected increase in futures price? To keep things simple, assume that the storage fee is paid up front and there is no variation in the futures price until maturity.

per contract:

time	now	six months out
buy fcoj	-\$20250 +15,000 lb. fcoj	
store fcoj	-15,000 lb. fcoj -\$3,600	+15,000 lb. fcoj
sell futures		-15,000 lb. fcoj +\$24,000
borrow	\$23,850	\$24,446.25
net		-\$446.25

not an arb

b. Suppose you can make a deal with a warehouse with excess capacity to rebate half of the storage fee so the effective cost of storage is \$300/month. Is there an arb in this case?

per contract:

time	now	six months out
buy fcoj	-$20250 +15,000 lb. fcoj	
store fcoj	-15,000 lb. fcoj -$1,800	+15,000 lb. fcoj
sell futures		-15,000 lb. fcoj +$24,000
borrow	$22,050	$22,601.25
net		+$1,398.75

an arb, thanks to the discounted storage

c. Given that we set up the arbitrage with the storage cost of $300/month, and that after three months the spot price is 155¢/pound and the futures price is still 160¢/pound. Should you unwind the position or leave it until maturity? (To keep things simple, assume that you can get a refund at month three on the remaining three months' storage, and you can also collect the variation in the futures contract so far at month three.)

per contract with exit at the three-month point:

time	now	three months out
buy fcoj	-$20250 +15,000 lb. fcoj	
store fcoj	-15,000 lb. fcoj -$1,800	+15,000 lb. fcoj $900 (refund)
sell fcoj		-15,000 lb. fcoj +$23,250
unwind futures		
borrow	$22,050	$22,325.62
net		$1,824.38

obviously, we prefer $1,824.38 three months out to $1,398.75 six months out, so we should unwind the position. The remaining increase in price of 5¢/pound does not cover the storage cost of 3 months $\times$ 300 dollars/month /15000 pounds = $.06 = 6¢/pound, let alone the interest cost of maintaining the position.

Note: There is never any cash flow for entering the futures contract. In this case there is never any other cash flow because the futures price hasn't changed.

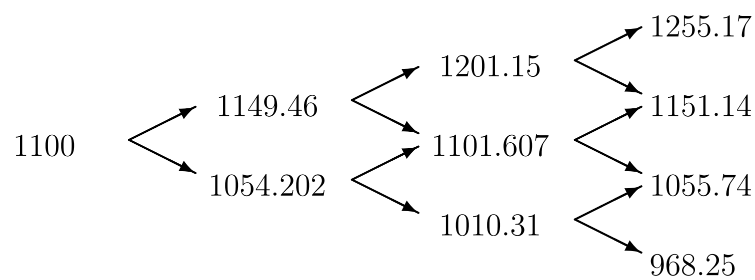
(extra for experts) How does the analysis change if we collect/pay the variation as the futures price goes up or down over the six months? In particular, you have to have the ability to pay whatever variation payments are due on the futures contracts. Suppose you provide for this using a line of credit — how big would it have to be?

(extra for experts: a little more work) What happens if we have to keep margin in cash (a non-interest-bearing account) according to the exchange's rules?

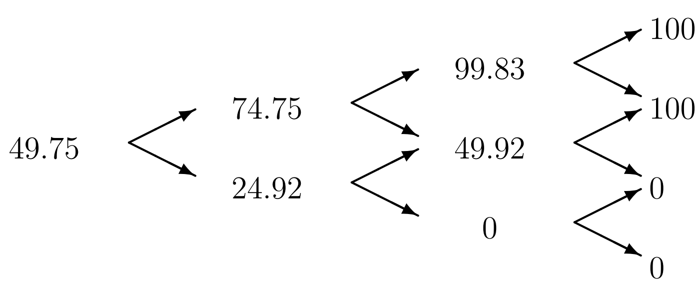
3. A market index is worth 1100 today and has a volatility of 15%/year. Assuming no dividends, evaluate a European digital option maturing three months from now paying $100 if the index exceeds 1105 and zero otherwise. Assume the risk-free rate is 2%. Evaluate the option using a three-period binomial.

$$\Delta t = 1/12$$
$$r = 1 + .02 * \Delta t = 1.0017$$
$$u = 1 + .02 * \Delta t + .15\sqrt{\Delta t} = 1.0450$$
$$d = 1 + .02 * \Delta t - .15\sqrt{\Delta t} = .9584$$
$$\pi^* = 1/2$$

Stock



Digital option:



Or, noting it is in the money half times (of the eight paths, one has final price 1255.17 and three have final price 1151.14, and all paths have the same probability 1/8), the option price is $0.5 * 100/1.02^3 = 49.75$.

(extra for experts) Write a computer program or use a spreadsheet to evaluate the digital option using 90 periods (approximately daily time interval).