Tigers and Flies: Conflicts of Interest, Discretion and Expertise in a Hierarchy*

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Abstract

We build a stylized theoretical model of decision-making in a hierarchy. The model is motivated by the ongoing anti-corruption campaign in China, but the analysis should be applicable to the policing of conflicts of interest in many hierarchies in governments, firms and other organizations. Borrowing terminologies from China, we can choose to reduce available corruption for tigers (high-level potentially corrupt officials) and flies (low-level potentially corrupt officials). We have three main results. First, we should go after both tigers and flies, the stated goal in China. Second, fighting corruption and imposing stringent constraints are substitutes: doing either one reduces the value of the other. Third, fighting corruption and training the flies are complements: doing one increases the value of the other. Fighting corruption and training can be used together.

JEL classification:

Key Words: Tigers and Flies, Conflicts of Interest, Discretion, Expertise

1 Introduction

Managing conflicts of interest is a big challenge for hierarchies in governments and organizations. We can think of this as an agency problem as modeled by Ross (1973) and Holmstrom (1979), and it is consistent with Krueger's (1974) claim that rent-seeking behavior is an important source of inefficiency in developing countries. Inspired by the colorful language of the ongoing anti-corruption program in China, we present a stylized model of decision-making in a hierarchy with high-level agents (tigers) and low-level agents (flies), all of whom are conflicted. The agency problems can be ameliorated by fighting corruption (which we will take to mean the same as policing conflict of interest) or by imposing stringent constraints. We find that (1) if available corruption is large, we want to fight corruption for both tigers and flies, (2) fighting corruption and imposing stringent rules are substitutes in the sense that if we do one, the other is not so useful, and (3) fighting corruption and training the flies are complements in the sense that doing one is more useful if we do the other. Fighting corruption plus training the flies can move from a strict bureaucracy to a technocracy.

Our analysis is motivated by the large-scale anti-corruption campaign started by Xi Jinping after he became the President of China in 2012. The ongoing campaign has many features, including an austerity campaign, a public-relations campaign, and an actual anti-corruption campaign.[1] One feature of the anti-corruption campaign is to reduce available corruption at different levels, so-called going after both the "tigers" and the "flies."[2] Our model adopts the colorful terminology from the Chinese anti-corruption campaign. In the model, a tiger decides how much discretion to give the fly. The fly and the tiger have different objectives, but the fly has information the tiger does not have, and the objectives are somewhat aligned, so the tiger will give the fly some discretion. The choice of how much discretion reflects the tiger's rational anticipation of the fly's use of the discretion to help the

[1]The austerity campaign restricts the use of the public funds for food, drink, gifts and entertainment. The public-relation campaign restricts self-promoting announcements and ostentatious behaviors. For instance, there should not be a welcoming banner, red carpet, floral arrangement, or a grand reception celebrating official visits.

[2]习近平：要坚持"老虎"、"苍蝇"一起打。(Xi, Jinping: We must crack down on both "tigers" and "flies".)

tiger against the fly's use of the discretion in ways that hurt the tiger. When the fly's incentives are well-aligned with the tiger's, as when the fly has little available corruption, the tiger gives the fly a lot of discretion. Alternatively, when the incentives are not well-aligned, as when the fly has lots of available corruption, the tiger will constrain the fly to have little discretion. Given the level of abstraction in the model, it can be useful for understanding many countries with different political systems, only with a modest change in the interpretations of the social goals.

Corruption, the damage from corruption, and the fight against corruption take many forms. There could be bribery to approve a business application or deny one to a competitor, or to create a new regulation that protects an existing business from competition, which would be bad for consumers and the economy but good for the existing business. A clerk may have a job of checking that a form (say an application for starting a new business) has the necessary stamps indicating approval from various agencies. Ideally, this verification would come automatically if the necessary approvals have been obtained. But if there is corruption, the verification would come only if the clerk is given a dinner and passed money – in China this would be in a "red envelope" (红包—hong bao). Fighting corruption may take the form of making these activities illegal or more likely they are already illegal and fighting corruption means putting more effort into detecting corruption and increasing the penalties when caught. The main benefits of fighting corruption are obvious: fighting corruption reduces the inefficiency caused by the corruption. The main subtlety of this is that some of the effect of corruption is just a transfer, just like paying part of the salary of the corrupt manager. The cost of corruption is not the transfer itself, but is rather the distortion of giving preference to companies that are better able to pay bribes rather than companies that provide the largest social benefit.

The costs of fighting corruption include the direct cost of the resources spent on fighting corruption (for example the time of people assigned to investigate different parts of the economy) and possibly a number of indirect costs. For example, investigation of firms and individuals for corruption probably takes a lot more of their time than time of the investigators, and this must hurt output in the economy. Also, anxiety about the campaign is also costly even if it improves incentives. And, it may not even improve incentives, because it may cause a sort of paralysis, for example, if officials do not approve any innovative firms because they are afraid of the appearance of corruption, especially if anything goes wrong. All of these different effects are interesting, but we abstract from the details so we can look at corruption at a high level. In our model, there are choices that are best for society, and the preferred choices that are best for various agents are different because of the available corruption. Our model is a reduced form in which reducing corruption is costly. For our later results, we also include the possibility of training low-level managers so they have expertise. In our model, we abstract from the details of what are the optimal choices and what is the design of the enforcement mechanism and penalties through which available corruption is reduced and cost is generated.

Our theoretical model looks at fighting corruption from the perspective of three different players: society, a tiger, and a fly. Preferences for society are hard to agree on in practice but simple in our model as the expected quadratic deviation from an exogenous random ideal point. In China, this ideal point might be determined by the Communist Party's assessment of what is best for society. In a western democracy, it might be some economically efficient benchmark. We are looking for results that are not dependent on the interpretation of the social ideal point or the government's political structure. Discretion in our model is not freedom or democracy; rather, discretion is only granted to the extent that incentives are there to serve the goals of people higher in the government hierarchy.

Both the tiger and the fly have their own preferences for deviation from what is socially optimal, so we can talk separately about fighting corruption of the tiger and of the fly. We abstract from the detailed mechanics of the anti-corruption campaign, including the exact process for identifying and punishing corruption, and the nature of the costs of the various elements of the campaign. Instead, we use a reduced form in which the policy variable for fighting the tiger's corruption is the standard deviation of the difference between tiger's ideal point and the social optimum. For each degree of available corruption (for the tiger and for the fly), there is some cost (in units of social welfare) to society of fighting corruption. The cost to society of reducing the availability of corruption is given by a pair of cost functions, one for the tiger's corruption and one for the fly's corruption. The cost functions are reduced forms in the sense that they specify the loss of social welfare for each choice of availability without modeling, explicitly where the reduction and cost come from. Our model does not specify whether the reduced available corruption comes from an increase in penalties, or an increase in the probability of being caught, or to fewer opportunities because potential counterparties are afraid of being caught. Only the fly has any information, in the form of a noisy signal of what the tiger wants. Everyone has the same priors on the distribution of the random variables in the model, and everyone knows the value of the policy variables, which are the extent of fighting the tiger's corruption, the extent of fighting the fly's corruption, and the expertise of the fly. We assume that society's ideal point, the two deviations, and the noise in the fly's information are independent normal variates.[3] This structure of the conflict provides a rich model but controls the complexity of the algebra.

Although this is an agency problem in that the tiger and the fly are making decisions but their incentives may not be aligned with society, we do not model this using an agency problem with optimal incentive contracting as in Ross (1973) or Holmstrom (1979). Instead, we think of the tiger and the fly as being separated by at least several layers of hierarchy and therefore the tiger does not have direct control over compensation, or perhaps more to the point, the time and information needed to construct a full incentive contract. Instead, the tiger has limited control over the fly through rule-setting that imposes constraints on the fly's actions. Because the fly has superior information not available to the tiger and some common interests, the tiger wants to give the fly at least some discretion. When the available corruption for the fly is scarce (due to the fighting corruption), the fly's interests are closely aligned to the tiger's interests, and the tiger will choose to give the fly a lot of discretion because the fly will make choices similar to what the tiger would choose.

The policy choices in the model are (1) how much to fight corruption of the flies, (2) how much to fight corruption of the tigers, and (3) how much training to give the flies. We model fighting corruption as a choice of

[3] Our original model allowed the fly's available corruption to be correlated with society's idea point. This could happen if part of the corruption opportunities of low-level managers might involve a bribe to allow socially beneficial competition to a big company. We do not think this sort of available corruption is the main thing going on (and if significant it would be shut down by the high-level managers anyway), so we do not include this in our model.

the standard deviation of available corruption opportunities, and we model training the flies as the choice of the standard deviation of the flies' signal of what the tigers want.

Our first theoretical result shows it is essential to fight corruption for both tigers and flies, not just for the flies. If available corruption is large, it might be tempting to fight corruption for the flies and leave the tigers alone, since the tigers are powerful and can fight back. However, reducing flies' available corruption helps tigers' corrupt goals more than any social goals, and consequently fighting the flies' corruption makes society worse off. Our second result finds that fighting corruption and stringent rules are substitutes, and the full benefits of reducing available corruption come if we relax rules. Probably it is tempting to impose more stringent rules at the same time when we intensively fight corruption, thinking we are clamping down on everything. But, our model illustrates that stringent rules reduce the benefit of fighting corruption, and that it would be optimal to accompany fighting corruption with a relaxation of stringent rules. Our third theoretical result shows that fighting corruption and enhancing expertise are complements. If the available corruption is high, it makes sense to constrain flies a lot, which neutralizes flies' expertise. With low available corruption and high expertise, the economy can flourish.

This paper is complementary to existing theoretical work on conflicts of interest in hierarchies. One interesting theoretical paper is Dewatripont et al. (1999), which looks at incentives of government officials who pursuing multiple missions. In their paper, increased incentives are endogenized, which come from professionalization and specialization of the officials. In our model, increased incentives are exogenized, which come from policing conflicts through fighting corruption or setting stringent rules. Another interesting paper, Prendergast and Topel (1996), shows that principal's preferences can be influenced by the agent due to the tight connection between these two levels. However, it is less likely to happen in our analysis because we assume high-level manager and low-level manger are separated by several levels in the hierarchy. Our model is also significantly different from the model of Bénabou and Tirole (2006), which interprets social preference as individual prosocial behavior, such as individual degree of altruism and greed on social reputation. In our paper, social preference is determined by the people of higher level in a hierarchy.

The paper is organized as follows. Section 2 presents and solves a theoretical model of society, the tiger and the fly. Section 3 shows that when the available corruption are great, we should crack down on both tigers and flies to be socially beneficial. Section 4 shows strict rules and fighting corruption are substitutes. Section 5 shows training flies and fighting corruption are complements. Section 6 closes the paper and summarizes our results.

2 Society and Equilibrium with Tigers and Flies

In this section, we present a theoretical model of conflicts in a hierarchy. We abstract from the hierarchy and consider two levels: the tiger and the fly. Probably it is useful to think of these two levels as being separated by several levels in a hierarchy, so the connection between the two is not so tight. We assume that the tiger has different objectives from the society's because of the the available corruption. The fly is the tiger's subordinate and consequently his preferences are similar to the tiger's, but he also has his own objective due to the available corruption. The tiger has imperfect control over the fly, executed by issuing rules that determine how much discretion the fly has. This limited control is why we think it makes sense to think of the tiger as being several levels above the fly in the hierarchy. The fly can collect extra rents (for both the fly and for the tiger) by deviating from the social optimum.

We assume the tiger and the fly jointly make a choice represented by a real number X, subject to constraints imposed by the tiger. The tiger does not have control over contracting as in the traditional agency literature following Ross (1973) and Holmstrom (1979). Instead, the tiger can only impose a constraint that the fly must choose X in some interval $[\underline{X}, \bar{X}]$. In other words, the endpoints of $[\underline{X}, \bar{X}]$ are chosen by the tiger.[4] The tiger knows that the fly has available corruption but does not have any information to

[4]As seems natural given the quadratic loss function and multivariate normal setting, it can be proven that if the tiger can choose any closed subset of $\Re$ as the restriction on X, the optimal choice would be a interval.

condition on, so $\underline{X}$ and $\bar{X}$ are constants. This limited degree of control is consistent with what is reasonable when the tiger and the fly are separated by at least several levels in the hierarchy.

The tension in the model comes from the fact that the tiger has the authority but the fly has information. To make this simple, we will assume that the fly's information is strictly better than the tiger's. We can think of this as conditioning on what the fly knows and computing payoffs given what both know, without the algebraic burden of modeling this explicitly. We assume that the society's ideal choice is given by the random variable S. The social welfare function is

$$
\begin{aligned}
(1) \quad W^S &= V^S - C_T(\gamma_T) - C_F(\gamma_F) - C_n(\sigma_n) \\
&= -E[(X-S)^2] - C_T(\gamma_T) - C_F(\gamma_F) - C_n(\sigma_n)
\end{aligned}
$$

where $S \sim N(0, \sigma_S^2)$ and V^S is social welfare ignoring costs. The tiger likes to be close to society's ideal point S, but can seek some private rents from deviating from society's ideal point. The level of fighting corruption depends on two policy choices, γ_T and γ_F. They represent the level of available corruption for the tiger and the fly, respectively, which can be reduced by the anti-corruption campaign. The fly's expertise level is inverse related with noise σ_n. $C_T(\gamma_T)$, $C_F(\gamma_F)$ and $C_n(\sigma_n)$ give the costs of fighting the tiger's corruption, fighting the fly's corruption, and training the fly, respectively. Smaller γ_T and γ_F indicate fewer available corruption, and smaller σ_n indicates better skill for the fly. The three cost functions $C_T(\cdot)$, $C_F(\cdot)$, and $C_n(\cdot)$ have similar properties. For $i =$ T, F and n, we assume that $\lim_{x\uparrow\infty} C_i(x) = 0$ and for $x > 0$, $C_i(x) > 0$, $C_i'(x) < 0$, and $C_i''(x) > 0$. For simplicity, we assume the tiger and the fly do not bear any of the cost of anti-corruption campaign, although what is important is the costs they bear are less than proportional to their benefit. The tiger's utility is

$$(2) \quad U^T = k_T E[-(X-S)^2 + 2\zeta_T(X-S)] = k_T[\gamma_T^2 - E((X-T)^2)]$$

where $\zeta_T \sim N(0, \gamma_T^2)$ is drawn independently of S, $k_T > 0$ and

$$(3) \quad T \equiv S + \zeta_T$$

and $T \sim N(0, \sigma_T^2)$ where

$$(4) \quad \sigma_T^2 = \sigma_S^2 + \gamma_T^2$$

We take $k_T = 1$, which is without loss of generality.[5] We interpret T as the tiger's ideal choice if the tiger knew S and ζ_T. In fact, neither S nor ζ_T is known by the tiger, or else the tiger's choice would depend on the information. For example, if the tiger knew both S and ζ_T, the tiger would choose $\underline{X} = \bar{X} = S + \zeta_T$ to force the fly to choose the tiger's ideal point. The rents extracted by the tiger from the deviation from the social ideal point S are given by $2\zeta_T(X - S)$.

The fly knows a signal I, which consists of T and noise term $\varepsilon_n \sim N(0, \sigma_n^2)$. We can write

$$(5) \quad I = T + \varepsilon_n$$

for $\sigma_I^2 = \sigma_T^2 + \sigma_n^2$. Smaller σ_n indicates better information, or equivalently more expertise, for the fly. If $\sigma_n = 0$, the fly knows T exactly, while in the limit $\sigma_n \uparrow \infty$, the fly knows nothing about T. We assume that ε_n, S and ζ_T are independent and joint normally distributed.

Given information I, the tiger's expected ideal choice is $\beta^I I$ where β^I is the coefficient from the linear regression

$$(6) \quad T = \beta^I I + \eta_I$$

Now, $\beta^I = cov(T, I)/var(I) = \sigma_T^2/\sigma_I^2 = \sigma_T^2/(\sigma_T^2 + \sigma_n^2)$, $var(\eta_I) = \sigma_T^2(1 - \beta^I)$, and $\beta^I I = E[T|I]$.

The fly likes the outcome to be near to the tiger's ideal point T, but can get some private rents from deviating from the tiger's ideal point. The fly's utility is

$$(7) \quad U^F = k_F E[-(X - T)^2 + 2\zeta_F(X - T)] = k_F[\gamma_F^2 - E((X - F)^2) - var(\eta_I)]$$

[5] Society and the tiger do not receive the same scale of benefits from reducing available corruption. However, taking $k_T = 1$ simplifies the algebra without affecting our results because multiplying the objective function by a constant does not change the optimal choice or ordering of alternatives.

where $\zeta_F \sim N(0, \gamma_F^2)$, is known by the fly. And based on (7), the fly's ideal choice of X absent constraints given I which is

$$(8) \quad F \equiv \beta^I I + \zeta_F,$$

A case can be made that ζ_F is correlated with S-T, since bribes could be for activities that produce benefits for society as well as for rent-seeking activities. We included these feature in our original analysis but assuming independence simplifies the algebra without changing the results significantly. So we assume simply that S, ζ_T, ζ_F and ε_n are independent and joint normally distributed, all having mean 0. We assume $k_F > 0$ and we set $k_F = 1$ for simplicity, which is without loss of generality for the same reason it was without loss of generality to set $k_T = 1$.

Now, $F \sim N(0, \sigma_F^2)$ where σ_F is the square root of

$$(9) \quad \sigma_F^2 = (\beta^I)^2 \sigma_I^2 + \gamma_F^2 = \beta^I(\sigma_T^2/\sigma_I^2)\sigma_I^2 + \gamma_F^2 = \beta^I \sigma_T^2 + \gamma_F^2$$

because information I and ζ_F are drawn independently. The fly knows I and ζ_F, but the tiger only knows the joint distribution of these variables. The privacy of the fly's superior information prevents the tiger from forcing the fly to choose X=I. The rents extracted by the fly from the deviation are given by $2\zeta_F(X - T)$. Since ζ_F has mean zero, the tiger cannot anticipate the direction of the fly's preferred deviation.

Given joint normality, the conditional expectation of T given F is given by $\beta^T F$ which comes from a linear regression

$$(10) \quad T = \beta^T F + \eta_T,$$

where

$$(11) \quad \beta^T = \frac{cov(F, T)}{var(F)} = \frac{\beta^I \sigma_T^2}{\sigma_F^2} = \frac{\beta^I(\sigma_S^2 + \gamma_T^2)}{\beta^I(\sigma_S^2 + \gamma_T^2) + \gamma_F^2} = \frac{1}{1 + \gamma_F^2/(\beta^I \sigma_T^2)}$$

and $\eta_T \sim N(0, \sigma_T^2(1 - \beta^I \beta^T))$. There is no constant term in the regression because F and T both have mean zero. The regression coefficient β^T, a number between 0 and 1, can be interpreted as the degree of alignment of the fly's incentives with the tiger's. We can see that the alignment is increasing in the fly's expertise (larger β^I or equivalently smaller σ_n) and decreasing in the level of the fly's available corruption (smaller γ_F). The alignment of incentives is best when $\beta^T = 1$ (when γ_F approaches 0), and worst when both β^T approach 0 (when γ_F approaches ∞).

Given the choice of $\underline{X}$ and $\bar{X}$ by the tiger, the fly's optimal response X is the projection of F on $[\underline{X}, \bar{X}]$, given by

$$(12) \quad X = \pi(F, \underline{X}, \bar{X}) = \begin{cases} \underline{X}, & \text{if } F < \underline{X}; \\ F, & \text{if } \underline{X} \leq F \leq \bar{X}; \\ \bar{X}, & \text{if } \bar{X} < F. \end{cases}$$

The first order condition for maximizing the tiger's utility (2), derived in Appendix B, implies that

$$(13) \quad \begin{cases} \underline{X} = \sigma_F \underline{x}(\beta^T) \\ \bar{X} = \sigma_F \bar{x}(\beta^T) \end{cases}$$

where $\bar{x}(\beta^T)$ is the solution of $\beta^T = \bar{x}N(-\bar{x})/n(\bar{x})$, and $\underline{x}(\beta^T) = -\bar{x}(\beta^T)$.

We do not know how to solve this explicitly for $\bar{x}$ given β^T, but almost just as useful, this is a parametric solution for β^T in terms of $\bar{x}$. The optimal $\bar{x}$ is increasing in β^T, with $\bar{x}$ moving from 0 to ∞ as β^T varies from 0 to 1, as shown in Appendix B.

3 Both Tigers and Flies

In this section, we show that when available corruption is high, fighting the fly's corruption (γ_F) without fighting the tiger's corruption (γ_T) makes society worse off.[6] In other words, when the tiger and the fly both have large available corruption, we should crack down on the fly and the tiger together to be socially beneficial. A second result in this section shows that the tiger

[6]Note: We are not so interested in fighting the tiger's corruption without fighting the fly's corruption. It is hard to fight the tiger's corruption because tigers are powerful and can fight back.

always prefers to fight the fly's corruption more than is socially optimally, because the tiger receives more than proportionally the benefits but bears less than proportionally the costs from fighting the fly's corruption.

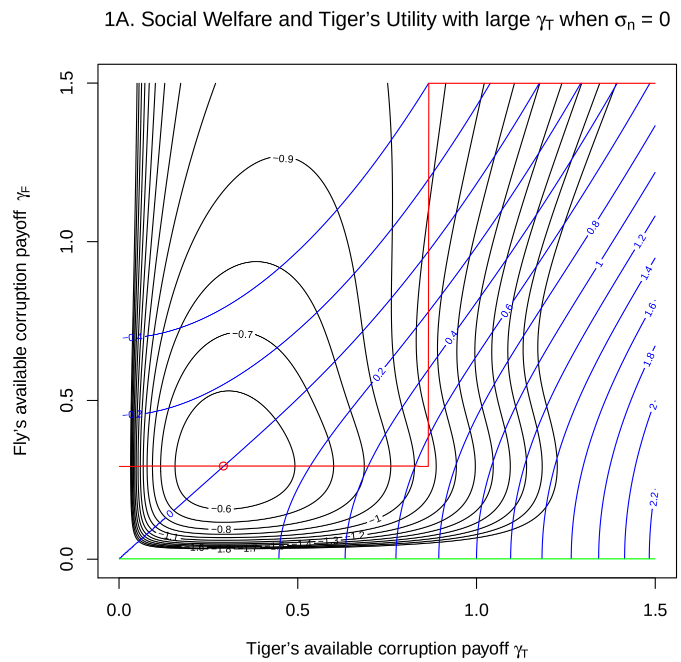
We present the results of this section first in figures, and then in formal theorems. Figures 1A and 1B show how social welfare (black curves) and the tiger's utility (blue curves) vary with different levels of the available corruption (γ_T and γ_F), for the fixed levels of noise σ_n ($\sigma_n = 0$ in Figure 1A and $\sigma_n = 1.5$ in Figure 1B). The contours of social welfare show that various degrees of reducing available corruption lead to different impacts on social welfare. The red dot denotes the social optimal. The contours of the tiger's utility show that the tiger is always better off when γ_T increases and γ_F decreases. When the tiger and the fly both have large available corruption (large γ_T and γ_F), fighting the fly's corruption (γ_F) without fighting the tiger's corruption (γ_T) makes society worse off because it aids the tiger's corruption. As we can see that social welfare contours are backward bending in the upper right region of Figures 1A and 1B. Formally,

Theorem 3.1. *We should crack down on both the tiger and the fly together and not just on the fly. If γ_F and γ_T are large enough, fighting the fly's corruption (through reducing γ_F) reduces social welfare, even ignoring the cost of fighting corruption.*

Proof. See Online Appendix C.1.
Sketch: The idea is to show that the total derivative of social welfare with respect to γ_F is positive, which says decreasing γ_F reduces social welfare.

$$\text{(14)} \quad \frac{dW^S}{d\gamma_F} = \frac{\partial V^S}{\partial \sigma_F}|_{\underline{X},\bar{X}} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial V^S}{\partial \underline{X}}|_{\sigma_F,\bar{X}} \frac{d\underline{X}}{d\gamma_F} + \frac{\partial V^S}{\partial \bar{X}}|_{\sigma_F,\underline{X}} \frac{d\bar{X}}{d\gamma_F} - \frac{dC_F(\gamma_F)}{d\gamma_F}$$

The first term on the right-hand side is the beneficial direct effect of giving the fly better incentives and is shown to be negative. The next two terms are harmful indirect effects of giving the fly more discretion, which are positive. The final term is the marginal cost of fighting the fly's corruption. When γ_F and γ_T are large enough, the harmful effects of increased discretion dominate the beneficial term, and the cost term makes it worse, so fighting the fly's corruption reduces social welfare, i.e. $dW^S/d\gamma_F > 0$. ■



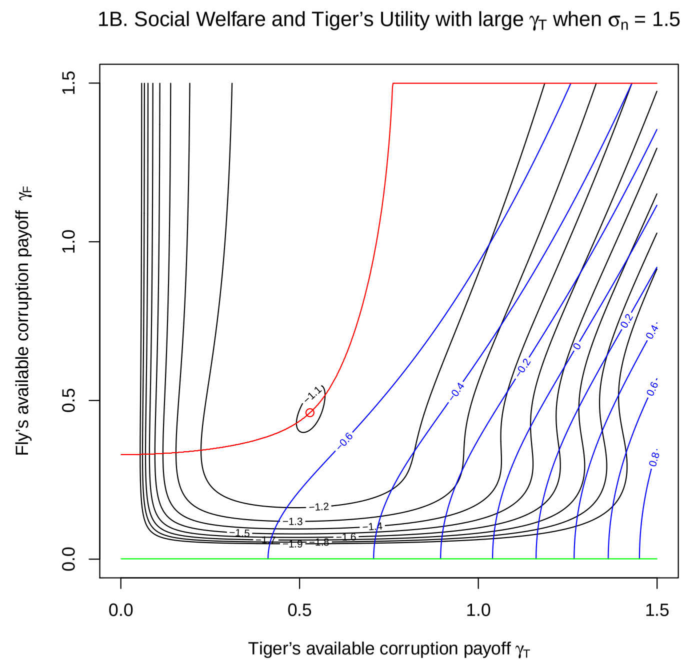


Figure 1A shows that we should go after both the tiger and the fly to benefit society. It is because when the tiger and the fly have large available corruption (large γ_T and γ_F), fighting fly's corruption γ_F without fighting tiger's corruption makes society worse off (black curves). Increasing tiger's available corruption γ_T and reducing fly's available corruption γ_F always makes the tiger better off (blue curves). Figure 1B shows, when comparing optimal levels of reducing fly's available corruption for the tiger (green solid line) and for society (red solid line), the tiger always has the tendency to choose more stringent level of fighting fly's corruption than is socially optimal. It is because the incentives are better aligned, and the tiger receives higher proportion of the benefits but bears less than proportion of the costs.

We also show the optimal levels of reducing fly's available corruption γ_F for the tiger (green solid line) and for society (red solid line).[7] For any level of γ_T, the tiger always wants a zero level of fly's available corruption ($\gamma_F = 0$), measured by the horizontal line. It is because reducing γ_F always makes the tiger better off, and the tiger gets more than proportionally the benefits but bears less than proportionally the costs (literally none of the cost in our model). However, for each level of γ_T, society always wants some positive level of the fly's available corruption ($\gamma_F > 0$) because society bears all the costs of fighting corruption ($C_F(\gamma_F)$ and $C_T(\gamma_T)$). Formally,

Theorem 3.2. *The tiger always prefers to fight the fly's corruption more than is socially optimal. In particular, given σ_S, γ_T and σ_n, reducing the fly's corruption always makes the tiger better off and therefore, the tiger wants the fly to have no available corruption ($\gamma_F = 0$), but society is better off with some positive level of the available corruption ($\gamma_F > 0$).*[8]

Proof. See Online Appendix C.2.
Sketch: The idea is to show that the total derivative of the tiger's utility with respect to γ_F is negative, which says increasing γ_F decreases the tiger's utility.

$$(15)\quad \frac{dU^T}{d\gamma_F} = \frac{\partial U^T}{\partial \sigma_F}\Big|_{\underline{X},\bar{X}} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial U^T}{\partial \underline{X}}\Big|_{\sigma_F,\bar{X}} \frac{d\underline{X}}{d\gamma_F} + \frac{\partial U^T}{\partial \bar{X}}\Big|_{\sigma_F,\underline{X}} \frac{d\bar{X}}{d\gamma_F}$$

The first term on the right-hand side is the marginal direct effect of making more corruption available to the fly, and is shown to be negative. The next two terms are the indirect effects of giving the fly more discretion, which are zero by the envelope theorem. I.e., the tiger can choose $\underline{X}$ and $\bar{X}$ optimally in response to γ_F, so $\partial U^T/\partial \underline{X} = \partial U^T/\partial \bar{X} = 0$. Therefore, reducing γ_F makes the tiger better off, i.e., $dU^T/d\gamma_F < 0$. Since there is no cost of reducing γ_F in (2), the benefits come at no cost to the tiger in (15). Therefore, the tiger's optimal choice is the lowest feasible value $\gamma_F = 0$. However, setting $\gamma_F = 0$ is

[7]Note: These levels are the best in the range in the graph, and values at the top should really be at a larger value, perhaps at infinity.

[8]Note: The tiger wants to reduce the fly's available corruption γ_F to zero because the tiger does not bear any of the cost. If the tiger bore a fraction of the cost but less than proportional to the tiger's share of the benefits, the tiger would still want to fight corruption for the fly more than is socially optimal.

not socially optimal, since society does bear the cost in social welfare (1) and $C_F(\gamma_F) \to \infty$ when $\gamma_F \to 0$, while the other terms in (1) are non-positive. Therefore, society prefers some $\gamma_F > 0$. ■

4 Information and Discretion

In this section, we show that fighting corruption and imposing stringent rules are substitutes. Since discretion is chosen by the tiger, our analysis of this result takes the tiger's perspective.[9] In choosing the level of discretion, the tiger would like the fly to use the fly's private information for their mutual benefit. However, when the fly's available corruption is high, the fly will not use the information in the way the tiger would like. Other things equal (in a way we will make precise), the tiger's optimal choice of $\bar{X}$ is decreasing in γ_F. This is the sense in which fighting corruption and stringent rules are substitutes. If the fly's available corruption is large (because there is no fighting of the fly's corruption), stringent rules are useful. Alternatively, if the fly's available corruption is small (because there is effective fighting of the fly's corruption), stringent rules mostly keep the fly from using private information to do what the tiger wants.

A second result in this section shows that increasing the fly's available corruption γ_F makes the fly better off when γ_F is small, but worse off when γ_F is large. Increasing γ_F has two effects. It gives the fly more profits given $\underline{X}$ and $\bar{X}$, but it also induces the tiger to shrink the interval $[\underline{X}, \bar{X}]$ in anticipation of the fly deviating more from what the tiger wants, which reduces profits for the fly. When γ_F is small, discretion is large, the first effect dominates, and the fly profits from taking advantage of the increased available corruption. When γ_F is large, the second effect dominates, and the fly is worse off when γ_F increases, because the main effect is from the reduction of discretion.

We present the results of this section first in figures, and then in formal

[9]Note: If we assumed (artificially) that society chooses $\underline{X}$ and $\bar{X}$ given γ_F, γ_T and σ_n, and that we fight corruption of tigers and flies together, we could derive a similar result from society's perspective.

theorems. Figure 2A shows that the alignment of incentives β^T decreases in relative available corruption γ_F/σ_T, and decreases in the relative noisiness σ_n/σ_T of the fly's signal. In other words, alignment of incentives is the best when the fly has few corruption opportunities and high expertise. Figure 2B shows that the fly's relative discretion $\bar{X}/\sigma_F$ increases in alignment of incentives β^T. In other words, discretion is the largest when the alignment of incentives approaches one. When the relative available corruption γ_F/σ_T is small, better aligning the interests for the fly and the tiger (larger β^T in Figure 2A), so the tiger gives more discretion to the fly (larger $\bar{X}/\sigma_F$ in Figure 2B). When the relative available corruption γ_F/σ_T is large, anticipating that the fly will collect more rents and care less about the tiger's ideal choice (smaller β^T in Figure 2A), the tiger gives the fly a very small range of X to choose (smaller $\bar{X}/\sigma_F$ in Figure 2B).

Figure 3A shows that fighting the fly's corruption γ_F always makes the tiger better off, and the tiger's utility is the highest at the optimal level of $\bar{X}$. When the fly has great available corruption (large γ_F, say $\gamma_F = 1.4$ in Figure 3A), the incentives between the tiger and the fly are badly aligned (implying smaller β^T). Therefore, small discretion ($\bar{X} = 0.3$) is optimal because the fly cares more about the rents collected from corruption than the tiger's preference. We see this in Figure 3A that: when $\gamma_F = 1.4$ (large), the tiger's utility is larger when $\bar{X} = 0.3$ (small) than when $\bar{X} = 0.5$, 1 or 2. When the fly has small available corruption (small γ_F, say $\gamma_F = 0.2$ in Figure 3A), the incentives between the tiger and the fly are well aligned (implying larger β^T). Therefore, large discretion ($\bar{X} = 2$) is optimal due to the benefits from delegating the jobs to the agent with better decision making. We see this in Figure 3A that: when $\gamma_F = 0.2$ (small), the tiger's utility is larger when $\bar{X} = 2$ (large) than when $\bar{X} = 0.3$, 0.5 or 1. Formally,

Theorem 4.1. *Fighting the fly's corruption and imposing stringent constraints on the fly are substitutes from the perspective of the tiger (who is the person choosing the level of discretion). In particular, $d\underline{X}/d\gamma_F > 0$ and $d\bar{X}/d\gamma_F < 0$.*

Proof. See Online Appendix D.1.
Sketch: The idea is to show that the total derivative of discretion with respect

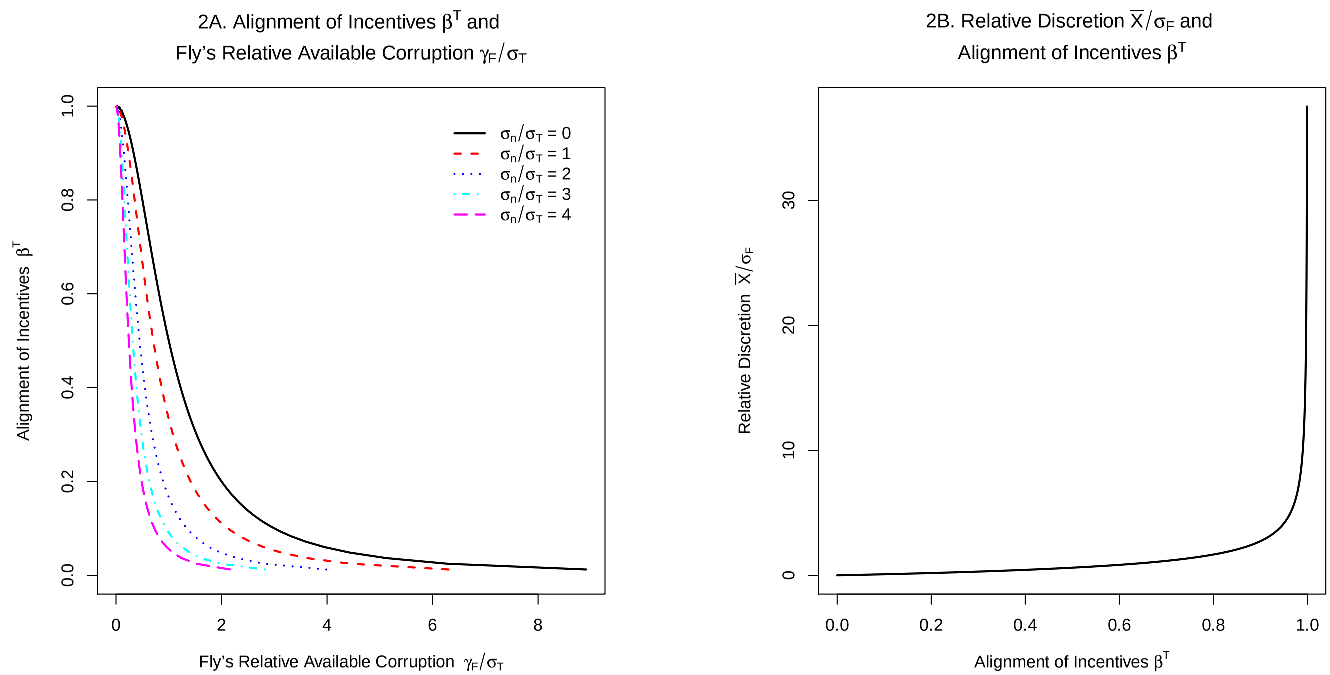


Figure 2A shows, given σ_T^2, having more relative available corruption (higher γ_F/σ_T) reduces the alignment of incentives (β^T) between the fly and the tiger (absent any constraint). γ_F/σ_T is the scale of profitability for the relative available corruption. σ_n/σ_T is the level of relative noise. β^T is the regression coefficient of the tiger's ideal point T on the fly's ideal point F. The tiger chooses the range of $[\underline{X}, \bar{X}]$ to constrain the fly's choices of X. Figure 2B shows, more discretion is given to the fly when the alignment of incentives is improved (larger β^T). The optimal interval $[\underline{X}, \bar{X}]$ is symmetric around 0, i.e. $\underline{X} = -\bar{X}$ in equilibrium and $\bar{X} > 0$. Larger $\bar{X}/\sigma_T$ corresponds to more discretion given to the fly. In these two figures, the larger γ_F/σ_T, the worse the alignment of incentives (smaller β^T) between the tiger and the fly, and the smaller the discretion granted, i.e., the more the tiger constrains the fly.

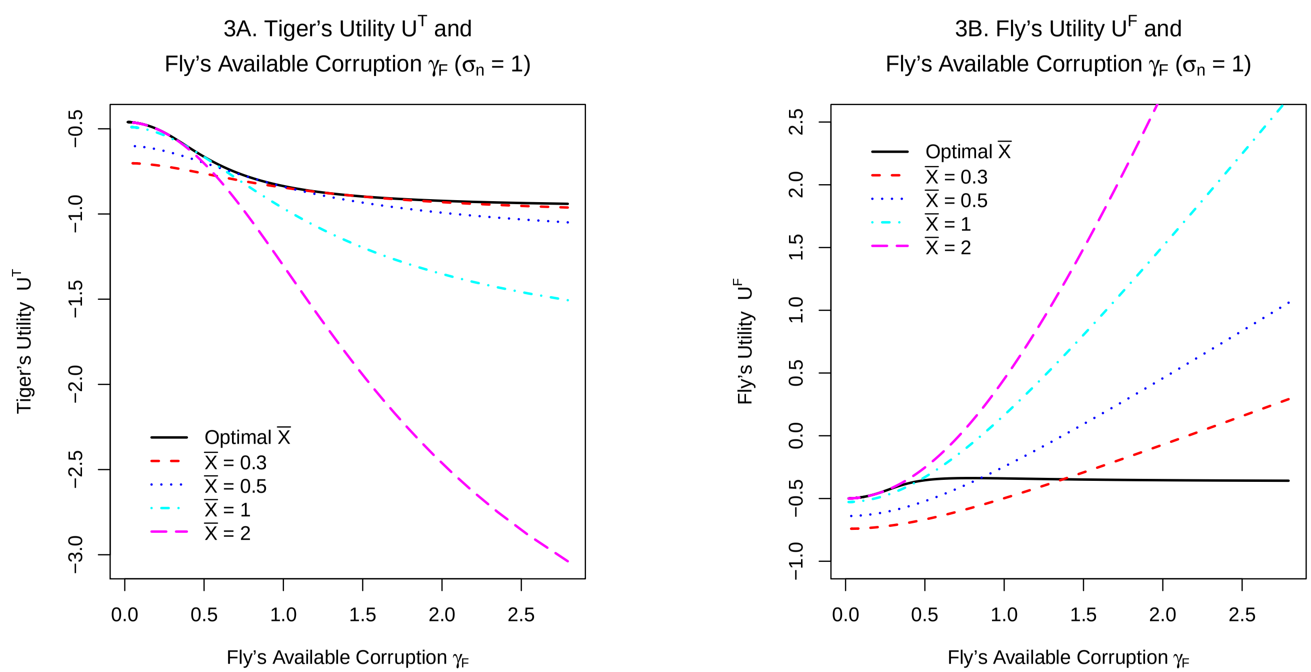


Figure 3A shows that, given $\bar{X}$, the tiger is always better off if the available corruption γ_F for the fly is reduced because this aligns their incentives better. The tiger can further exploit the improved alignment of incentives by giving the fly more discretion. When γ_F is small, large discretion to the fly makes the tiger better off because more jobs are delegated to the agent with better information. When γ_F is large, large discretion to the fly makes the tiger worse off because the fly cares more about the rents from corruption than the tiger's objective. Figure 3B shows that at the optimal $\bar{X}$, reducing fly's available corruption makes the fly better off when γ_F is small and worse off when γ_F is large. When γ_F is small, fighting corruption makes the fly worse off because benefits from available corruption are larger than the increases in the discretion. When γ_F is large, fighting corruption makes the fly better off because the increase in the discretion is more important than the rents collected from the available corruption.

to γ_F is negative, which says increasing γ_F decreases discretion.

$$(16) \quad \frac{d\bar{X}}{d\gamma_F} = \frac{\partial \bar{X}}{\partial \sigma_F}|_{\beta_T} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial \bar{X}}{\partial \beta^T}|_{\sigma_F} \frac{d\beta^T}{d\gamma_F}$$

The first term is positive because $\partial \bar{X}/\partial \sigma_F = \bar{x}(\beta^T)$ and σ_F increases in γ_F when β^T is fixed. The second term is negative because $\bar{X}$ increases in β^T but the alignment of incentives β^T decreases in γ_F when σ_F is fixed. When γ_F increases, the dominant effect is the reduction in the alignment of incentives β^T. That is, $\bar{x}(\beta^T)$ falls relatively more than σ_F increases. Similar, we have the symmetric result for $\underline{X}$, because the optimal $\underline{X} = -\bar{X}$. ■

Figure 3B shows that at the optimal $\bar{X}$, reducing the fly's available corruption γ_F makes the fly worse off when γ_F is small and better off when γ_F is large. When γ_F is small enough, anti-corruption makes the fly worse off because the reduction in available corruption is more important than the increase in discretion (which is already large). When γ_F is large enough, anti-corruption makes the fly better off because the reduction in available corruption is less important than the additional discretion (which is almost zero). For fixed levels of $\bar{X}$, given the same availability of the corruption opportunities γ_F, the fly is always better off with more discretion (larger $\bar{X}$). Formally, we have

Theorem 4.2. *Increasing the fly's available corruption γ_F makes the fly better off when γ_F is small and worse off when γ_F is large. The direct impact of increasing γ_F benefits the fly because more opportunities are available. There is also an indirect effect that hurts the fly, which comes from the tiger trusting the fly less and giving the fly less discretion. When γ_F is small enough, the direct effect dominates, and fighting corruption makes the fly worse off. When γ_F is large enough, the indirect effect dominates, and fighting corruption makes the fly better off.*

Proof. See Online Appendix D.2.
Sketch: The idea is to show that the total derivative of the fly's utility with respect to γ_F is positive when γ_F is small, but negative when γ_F is large.

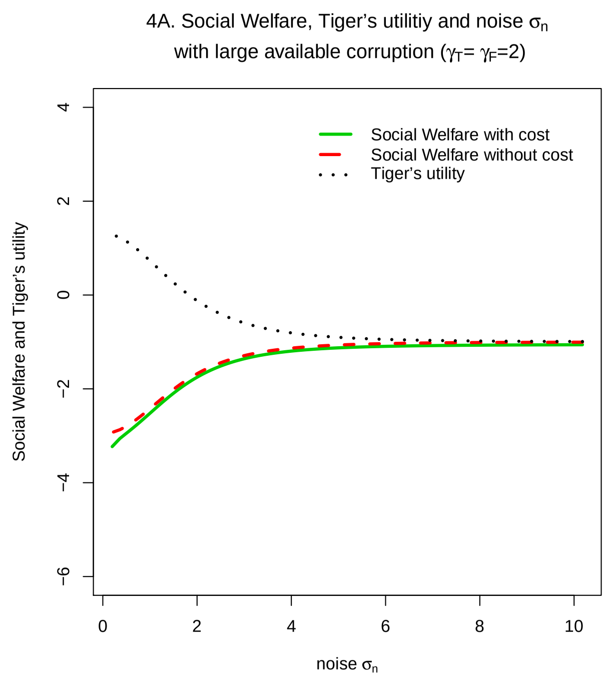
$$(17) \quad \frac{dU^F}{d\gamma_F} = \frac{\partial U^F}{\partial \sigma_F}|_{\underline{X},\bar{X}} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial U^F}{\partial \underline{X}}|_{\sigma_F,\bar{X}} \frac{d\underline{X}}{d\gamma_F} + \frac{\partial U^F}{\partial \bar{X}}|_{\sigma_F,\underline{X}} \frac{d\bar{X}}{d\gamma_F}$$

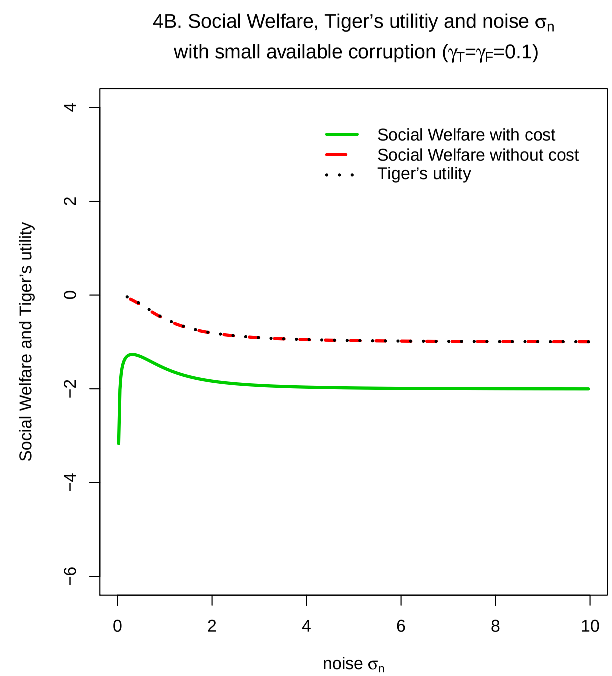
The first term on the right-hand side is the direct effect of increasing the fly's available corruption and is shown to be positive. The next two terms are the indirect effects of giving the fly more discretion. When γ_F is small enough, the direct effect is shown to dominate, and when γ_F is large enough, the indirect effect is show to dominate. ■

5 Expertise

In this section, we show that fighting corruption and enhancing expertise are complements. From society's perspective, when the available corruption for both the tiger and the fly is high, $\underline{X}$ and $\bar{X}$ will be close to zero, and expertise will not matter much. Stringent rules tend to neutralize expertise. If expertise is low, the benefit of improving the fly's incentives is small and fighting corruption probably will not be worth the cost. Conversely, if the fly's available corruption is small and expertise is high, the range of $[\underline{X}, \bar{X}]$, will be chosen to be large and the expertise will have a big impact on the choice of X. In this case, training the fly and reducing the fly's available corruption benefits the tiger but may hurt society even ignoring the cost, because expertise helps the fly to do more of what the corrupt tiger wants. In general, the tiger bears more the benefits than the costs of training the fly, and would choose a higher level of training than society would.

We present the results of this section first in figures, and then in formal theorems. Figures 4A and 4B show how social welfare (with and without cost) and the tiger's utility change with noise σ_n. We show that training the fly (reducing σ_n) makes society worse off when the tiger has large available corruption, even ignoring the cost. However, training the fly always makes the tiger better off. It is because the tiger does not bear any cost and the fly is working for him. Even if the tiger bore some cost, we expect that the tiger would prefer to train the fly more than is socially optimal because the tiger's share of benefits are larger than the tiger's share of costs. In Figure 4A, when the tiger and the fly both have large available corruption (where $\gamma_T = \gamma_F = 2$), training that benefits the tiger hurts society because it helps the tiger to implement his corruption. In Figure 4B, when available corruption is scarce (where $\gamma_T = \gamma_F = 0.1$), training the fly makes society





In Figure 4A, when γ_F and γ_T are large, training that benefits the tigers hurts society because it helps the tigers to implement their corruption. In Figure 4B, when γ_F and γ_T are small, training the flies (to reduce σ_n) helps society if the costs can be justified. In this case, the main difference between society's preference and that of the tigers' is the cost of training. In both cases, the tigers always prefer training the flies because they do not bear the cost and the flies are working for them.

better off if the cost is not too high. In this case, the main difference between society's preference and that of the tiger is the cost of training. Formally, we have

Theorem 5.1. *Fighting corruption and training the fly are complements. If γ_F and γ_T are both large enough, training the fly (reducing σ_n) reduces social welfare, even ignoring the cost of training. However, if γ_F and γ_T are both small enough, training the fly improves social welfare ignoring the cost (but of course may not be justified if the marginal cost is high enough).*

Proof. See Online Appendix E.1.
Sketch: The idea is to show that the total derivative of social welfare with respect to σ_n is positive when γ_F and γ_F are both large, but negative when γ_F and γ_T are both small.

$$\frac{dW^S}{d\sigma_n} = \frac{\partial V^S}{\partial \sigma_F}|_{\underline{X},\bar{X}} \frac{d\sigma_F}{d\sigma_n} + \frac{\partial V^S}{\partial \underline{X}}|_{\sigma_n,\bar{X}} \frac{d\underline{X}}{d\sigma_n} + \frac{\partial V^S}{\partial \bar{X}}|_{\sigma_n,\underline{X}} \frac{d\bar{X}}{d\sigma_n} - \frac{dC_n(\sigma_n)}{d\sigma_n} \tag{18}$$

The first term on the right-hand side is the beneficial direct effect of giving the fly better training and is shown to be negative. The next two terms are the harmful indirect effects of giving the fly more discretion, which are positive. The final term is the marginal cost of training the fly. When γ_F and γ_T are small enough, the beneficial effect of training the fly dominates harmful terms, so training the fly increases social welfare if we are in the region where the marginal cost of training is not too high, i.e. $dW^S/d\sigma_n < 0$. ∎

Theorem 5.2. *Increasing the fly's expertise (training that reduces σ_n), always makes the tiger better off because the expertise gives the fly better information about what the tiger wants and aligns the fly's incentives better with the tiger's. Increasing the fly's expertise also generates indirect benefits to the tiger because the tiger optimally gives the fly more discretion so that more decisions are made by the agent with superior information.*

Proof. See Online Appendix E.2.
Sketch: The idea is to show that the total derivative of the tiger's utility with respect to σ_n is negative, which says decreasing σ_n increases the tiger's

utility.

$$(19) \quad \frac{dU^T}{d\sigma_n} = \frac{\partial U^T}{\partial \sigma_F}|_{\underline{X},\bar{X}} \frac{d\sigma_F}{d\sigma_n} + \frac{\partial U^T}{\partial \underline{X}}|_{\sigma_n,\bar{X}} \frac{d\underline{X}}{d\sigma_n} + \frac{\partial U^T}{\partial \bar{X}}|_{\sigma_n,\underline{X}} \frac{d\bar{X}}{d\sigma_n}$$

The first term on the right-hand side is the direct effect of increasing the fly's noise and is shown to be negative. The next two terms are the indirect effects of giving the fly more discretion, which are zero by the envelop theorem. I.e., the tiger can choose $\underline{X}$ and $\bar{X}$ optimally in response to γ_F, so $\partial U^T/\partial \underline{X} = \partial U^T/\partial \bar{X} = 0$. Although the benefit of changing $\underline{X}$ and $\bar{X}$ is zero to first order (at the optimum), it does carry a higher order benefit, which is why $d\underline{X}/d\sigma_n > 0$ and $d\bar{X}/d\sigma_n < 0$. Therefore, training the fly always makes the tiger better off, i.e. $dU^T/d\sigma_n < 0$. ■

In another case (not shown), when the fly has more available corruption than the tiger does, training the fly does not have a significant impact on the society, because when the fly's available corruption is large, the tiger is not going to give the fly much discretion. The benefit to society, if any, will be less than the cost. Strict rules neutralize expertise. Furthermore, when the tiger has much more available corruption than the fly does, training the fly will cause a loss to society, because when the fly's available corruption is scarce, the tiger is going to give the fly more discretion. This induces the fly to do more of what the corrupt tiger wants, and makes society worse off. That is, even if the fly does not have large available corruption but the tiger has, training the fly is socially undesirable.

6 Conclusion

This paper builds a theoretical model of conflicts in a hierarchy using the colorful terminology of the ongoing anti-corruption campaign in China. The issues of balancing control with provision of incentives is more universal and arises in all governments. We focus on the true anti-corruption campaign and discuss effects of reductions in available corruption to society, tigers and flies. We have three main conclusions.

First, if tigers and flies have large available corruption, fighting the flies' corruption without fighting the tigers' corruption reduces social welfare. It is because reducing flies' available corruption helps the tigers to implement their corruption. Second, stringent rules and fighting corruption are substitutes, i.e., to take full advantage of the aligned incentives, a reduction of corruption opportunities should be accompanied by more discretion. Third, training flies and fighting corruption are complements, because flies who have large available corruption can not use expertise, since they are given very little discretion.

An effective fight against corruption at all levels can make society more efficient by aligning incentives and allowing the allocation of control rights to the people with the information needed to make decisions. The greatest improvement will occur if fighting of corruption is accompanied by efficient delegation of decision-making and the development of the appropriate level of expertise, moving from a strict bureaucracy to a technocracy that implements society's goals. The game model is applicable to planned economies, market economies, or firms and other organizations with appropriate interpretations of the players and social goals.

Appendix A Properties of the Normal Distribution

Lemma A1. *Let* $n(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$ *and* $N(x) = \int_{y=-\infty}^{x} n(y)\, dy$ *be the unit normal density and cumulative distribution functions, respectively. Thus,*
(a) $N'(x) = n(x)$, $n'(x) = -xn(x)$, $n''(x) = -n(x) + x^2 n(x)$;
(b) $N(x)$ *is an increasing 1-1 function mapping* $(-\infty, +\infty)$ *onto (0,1);*
(c) For $x < 0$, $g(x) \equiv N(x)+n(x)/x+N(x)/x^2$ *is an increasing 1-1 function mapping* $(-\infty, 0)$ *to* $(0, +\infty)$;
(d) For $x < 0$, $p(x) \equiv g(x)x^2/N(x)$ *is an increasing 1-1 function mapping* $(-\infty, 0)$ *to (0,1);*
(e) For $x > 0$, $f(x) \equiv xN(-x)/n(x)$ *is a monotonically increasing 1-1 function mapping* $(0, +\infty)$ *onto (0,1);*

(f) For $x > 0$,

$$N(-x) = \frac{n(x)}{x}\left\{1 - \frac{1}{x^2} + \frac{1*3}{x^4} + \cdots + \frac{(-1)^n * 1 * 3 * \cdots * (2n-1)}{x^{2n}}\right\} + R_n$$

where

$$R_n = (-1)^{n+1} * 1 * 3 * \cdots * (2n+1) \int_x^{+\infty} \frac{n(t)}{t^{2n+2}} dt$$

which is less in absolute value than the first neglected term.

Proof:
(a) simple algebra and calculus.

(b) N(x) is increasing and continuous because it is differentiable and $N'(x) = n(x) > 0$, and is therefore 1-1 from its domain $(-\infty, +\infty)$ to its range, which is (0,1) because n(x) is a probability density.

(c)

$$\frac{dg(x)}{dx} = \left[n(x) - n(x) - \frac{n(x)}{x^2} + \frac{n(x)}{x^2} - \frac{2N(x)}{x^3}\right] = -\frac{2N(x)}{x^3}$$

Therefore, $g(x)$ is increasing when $x < 0$. As $x \to -\infty$, $g(x) \to 0$ because each term goes to 0; As $x \to 0$, g(x) is determined by the term $N(x)/x^2$, so $g(x) \to +\infty$. Thus, for $x \in (-\infty, 0)$, $g(x) \in (0, +\infty)$.

(d)

$$g(x) = \frac{N(x)}{x^2} - \int_{y=-\infty}^{x} \frac{n(y)}{y^2}\, dy$$

$$p(x) = \left(\frac{N(x)}{x^2} - \int_{y=-\infty}^{x} \frac{n(y)}{y^2}\, dy\right)\frac{x^2}{N(x)} = 1 - \frac{1}{N(x)} \int_{y=-\infty}^{x} \frac{x^2 n(y)}{y^2}\, dy$$

Then

$$0 < \frac{1}{N(x)} \int_{y=-\infty}^{x} \frac{x^2 n(y)}{y^2}\, dy = 1 - p(x) < \frac{1}{N(x)} \int_{y=-\infty}^{x} \frac{y^2 n(y)}{y^2}\, dy = 1$$

Thus, for $x \in (-\infty, 0)$, $p(x) \in (0,1)$.

(e)

$$\begin{aligned} \frac{df(x)}{dx} &= \frac{d}{dx}\left(\frac{xN(-x)}{n(x)}\right) = \frac{N(-x)}{n(x)} - x + \frac{x^2 N(-x)}{n(x)} \\ &= \underbrace{\frac{x^2}{n(x)}}_{>0} \left(\underbrace{N(-x) + n(-x)/(-x) + N(-x)/(-x)^2}_{g(-x)>0} \right) > 0 \end{aligned}$$

Thus, $f(x)$ is strictly increasing.

When $x \to 0$, $f(x) \to 0$. When $x \to +\infty$, $N(-x) \to 0$, $n(x)/x \to 0$, thus

$$\lim_{x\to+\infty} \frac{N(-x)}{n(x)/x} = \lim_{x\to+\infty} \frac{dN(-x)/dx}{d(n(x)/x)/dx} = \lim_{x\to+\infty} \frac{-n(x)}{-n(x) - n(x)/x^2} = \lim_{x\to+\infty} \frac{1}{1 + 1/x^2} = 1$$

There, for $x > 0$, $f(x) \equiv xN(-x)/n(x)$ is a monotonically increasing 1-1 function mapping $(0, +\infty)$ onto (0,1).

(f) see Abramowitz and Stegun (1965), 26.2.12.

■

Appendix B Characterizing optimal $\underline{x}$ and $\bar{x}$

Since by assumption $k_T = 1$, the tiger's utility from (2) is

(20) $U^T = \gamma_T^2 - E[(X-T)^2]$

Now,

$$(21)\quad E[(X-T)^2] = E[(X-(\beta^T F+\eta_T))^2] = E[(X-\beta^T F)^2 - 2\eta_T(X-\beta^T F) + {\eta_T}^2]$$
$$= E[(X-\beta^T F)^2] + var(\eta_T)$$

$E[\eta_T(X-\beta^T F)] = 0$ because $E[\eta_T]=0$, the fly's choice of X is a function of F, and η_T and F are independent. From (12), (20) and (21),

$$(22)\quad U^T = \gamma_T^2 - E[(\pi(F,\underline{X},\bar{X}) - \beta^T F)^2] - var(\eta_T)$$
$$= \gamma_T^2 - \int_{\varphi=-\infty}^{\underline{x}} (\underline{x}\sigma_F - \beta^T\varphi\sigma_F)^2 n(\varphi)\,d\varphi - \int_{\varphi=\underline{x}}^{\bar{x}} (\varphi\sigma_F - \beta^T\varphi\sigma_F)^2 n(\varphi)\,d\varphi$$
$$- \int_{\varphi=\bar{x}}^{+\infty} (\bar{x}\sigma_F - \beta^T\varphi\sigma_F)^2 n(\varphi)\,d\varphi - var(\eta_T)$$

where $\varphi \equiv F/\sigma_F \sim N(0,1)$ and $\bar{x} \equiv \bar{X}/\sigma_F$. By Lemma A1(a), $n'(\varphi) = -\varphi n(\varphi)$, the tiger's first-order condition for $\bar{x}$ is

$$(23)\quad 0 = \frac{\partial}{\partial\bar{x}} E[-(\pi(\varphi\sigma_F,\underline{x}\sigma_F,\bar{x}\sigma_F) - \beta^T\varphi\sigma_F)^2]$$
$$= -2\sigma_F^2\int_{\varphi=\bar{x}}^{+\infty}(\bar{x}-\beta^T\varphi)n(\varphi)\,d\varphi = -2\sigma_F^2\left(\bar{x}\int_{\varphi=\bar{x}}^{+\infty} n(\varphi)\,d\varphi + \beta^T\int_{\varphi=\bar{x}}^{+\infty} n'(\varphi)\,d\varphi\right)$$
$$= -2\sigma_F^2\left(\bar{x}N(\varphi)|_{\bar{x}}^{+\infty} + \beta^T n(\varphi)|_{\bar{x}}^{+\infty}\right) = -2\sigma_F^2[\bar{x}N(-\bar{x}) - \beta^T n(\bar{x})]$$
$$= -2\sigma_F^2 n(\bar{x})\left[\frac{\bar{x}N(-\bar{x})}{n(\bar{x})} - \beta^T\right]$$

Similarly, the tiger's first-order condition for $\underline{x} \equiv \underline{X}/\sigma_F$ is

$$(24)\quad 0 = \frac{\partial}{\partial\underline{x}} E[-(\pi(\varphi\sigma_F,\underline{x}\sigma_F,\bar{x}\sigma_F) - \beta^T F)^2] = -2\sigma_F^2 n(\underline{x})\left[\frac{\underline{x}N(\underline{x})}{n(\underline{x})} + \beta^T\right]$$

Since the positive coefficient σ_F does not affect Equation (23), the optimal $\bar{x}$ is independent of σ_F (given β^T), i.e. $\bar{X} = \sigma_F\bar{x}$ where $\bar{x}$ is a constant solving Equation (23) given β^T. Thus,

$$(25)\quad \beta^T = \frac{\bar{x}N(-\bar{x})}{n(\bar{x})} = \frac{-\underline{x}N(\underline{x})}{n(\underline{x})}$$

Thus, Lemma A1(e) can be used to show that, 1) the optimal choices of $\underline{x}$ and $\bar{x}$ are determined uniquely by the first order conditions (23) and (24), and 2) the optimal $\bar{x}$ (respectively $\underline{x} = -\bar{x}$) is increasing (respectively decreasing) in β^T. From Lemma A1(e), Equation (23) has a unique solution given β^T, call it $\bar{x}(\beta^T)$. Furthermore, by Equation (23) and Lemma A1(e), $dU^T/d\bar{X} > 0$ when $\bar{x} < \bar{x}(\beta^T)$ and $dU^T/d\bar{X} < 0$ when $\bar{x} > \bar{x}(\beta^T)$. It implies that $\bar{x}(\beta^T)$ (or more precisely $\bar{X} = \sigma_F \bar{x}(\beta^T)$) is the tiger's unique optimal choice. The proof is similar that $\underline{X} = -\sigma_F \bar{x}(\beta^T)$ is also optimal. Furthermore, Equation (23) and Lemma A1(e) imply that $\bar{x}(\beta^T)$ is an increasing function.

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