Tigers and Flies: Discretion and Expertise in the Chinese Anti-corruption Campaign

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Abstract

We build a theoretical model of an anti-corruption campaign, such as the ongoing campaign in China. The anti-corruption campaign reduces corruption incentives of the tigers (high-level potentially corrupt officials) and flies (low-level potentially corrupt officials). The anti-corruption campaign should go after the tigers as well as the flies (the stated goal in China) to benefit society, because when both flies and tigers are very corrupt, reducing the corruption of the flies but not the tigers only aids tigers’ corruption. Strict rules and fighting corruption are substitutes. To take full advantage of the aligned incentives, a reduction of corruption should be accompanied by giving the flies more discretion. On the other hand, fighting corruption and training flies are complements, since there is no point in training flies if you don’t trust them anyway and put strict limits on their actions, and there is no point in reducing flies corruption if they have no expertise. Fighting corruption may start the path from a bureaucracy with strict rules to a technocracy with aligned incentives and more expertise.

JEL classification:
Key Words: Anti-corruption Campaign, China, Tigers and Flies, Information, Discretion, Expertise
1 Introduction

In 2012, a large-scale anti-corruption campaign was started by Xi Jinping after becoming the President of China. The ongoing campaign has many features, including an austerity campaign and a public-relation campaign, as well as an actual anti-corruption campaign.\(^1\) We model an actual anti-corruption campaign that reduces the availability of profitable corruption opportunities and works to align incentives with social goals. One feature of the anti-corruption campaign in China is to reduce the corruption opportunities at different levels, so-called going after both the “tigers” and the “flies”.\(^2\) In this paper, “tigers” are high-level potentially corrupt officials, and “flies” are low-level potentially corrupt officials. The tigers decide how much discretion to give the flies. This decision trades off flies’ ability to use their superior information to help the tigers against flies’ ability to use the discretion in pursuit of their own agendas. We have three main results. First, while it is relatively easy to fight corruption only for the flies, the anti-corruption campaign should crack down on both tigers and flies to be effective. Second, the benefits from reducing corruption comes through two channels: better alignment of incentives given the existing level of discretion, and even better choices given that discretion is increased. We can summarize this second point by saying that strict rules limiting discretion and fighting corruption are substitutes, since it may make sense to have either one or the other but not both. Third, when corruption is high, discretion is low and expertise is not useful. Once corruption is lower and discretion is increased, training to increase flies’ expertise is more useful. We can think of this third point as saying that fighting corruption and enhancing flies’ expertise are

\(^1\)The austerity campaign restricts the use of the public funds for food, drink, gifts and entertainment. The public-relation campaign restricts self-promoting announcements and ostentatious behaviors. For instance, there should not be a welcoming banner, red carpet, floral arrangement, or a grand reception celebrating official visits.

\(^2\)习近平：要坚持“老虎”、“苍蝇”一起打。(Xi, Jinping: We must crack down on both “tigers” and “flies”.)
complements, since each is most effective when you do the other one as well. Fighting corruption and enhancing flies’ expertise could help China to move from a bureaucracy in which the low-level officials are highly constrained and mostly concerned with following rules to more of a technocracy in which low-level officials have good incentives to do good things for society and the expertise and discretion needed to do them.

Our theoretical model looks at corruption from the perspective of three different players: society, a tiger, and a fly. Preferences for society are hard to agree on in practice but simple in our model as the expected quadratic deviation from a random ideal point. Both the tiger and the fly have their own preferences for deviation from what is socially optimal, so we can talk separately about fighting corruption of the tiger and corruption of the fly. We abstract from the detailed mechanics of the anti-corruption campaign, including the exact process for identifying and punishing corruption, and the nature of the costs of the various elements of the campaign. Instead, we use a reduced form in which the policy variable for fighting corruption of the tiger is the standard deviation of the difference between tiger’s ideal point and the social optimum. For each degree of corruption (for the tiger and for the fly) there is some cost (in units of social welfare) to society of reducing corruption. Only the fly has any information, in the form of a noisy signal of what the tiger wants. Everyone has the same priors on the distribution of the random variables in the model. Everyone knows the value of the policy variables, which are the extent of fighting the tiger’s corruption, the extent of fighting the fly’s corruption, and the expertise of the fly. To keep the algebra simple, we assume that society’s ideal point, the two deviations, and the noise in the fly’s information are independent normal variates.

Although this is an agency problem in that the tiger and the fly are making decisions but their incentives may not be aligned with society, we do not model this using an agency problem with optimal incentive contracting as in Ross (1973) or Holmstrom (1979). Instead, we think of the tiger and the
fly as being separated by at least several layers of hierarchy and therefore the tiger does not have direct control over compensation. Instead, the tiger has limited control over the fly through rule-setting that imposes constraints on the fly’s actions, and the policy variables are the two levels of corruption (the result of the anti-corruption campaign targeted at the tiger and at the fly) and the amount of expertise for the fly (the result of training). Because the fly has superior information not available to the tiger and some common interests, the tiger wants to give the fly at least some discretion. When corruption opportunities for fly are scarce (due to an anti-corruption campaign), the fly’s interests are closely aligned to the tiger’s interests, and the tigers will choose to give the fly a lot of discretion because the fly will make choices similar to what the tiger would choose.

Our first theoretical result shows it is essential to fight corruption for both tigers and flies, not just for the flies. If corruption is large, it might be tempting to fight corruption for the flies and leave the tigers alone, since the tigers are powerful and can fight back. However, reducing flies’ corruption opportunities helps tigers’ corruption more than any social goals and makes society worse off. We also show that tigers would like to impose a more severe anti-corruption campaign on flies than is socially optimal. This is because tigers get more than proportional benefits from the anti-corruption campaign and bear less than proportional costs. Our second result finds that part of the benefit of anti-corruption campaign for flies is the direct impact from aligning incentives and part is the indirect benefit from giving the flies with better incentives, or equivalently, more discretion. In other words, the anti-corruption campaign, and strict rules are substitutes, and the full benefits of an anti-corruption campaign comes if we relax rules. Probably it is tempting to impose stricter rules at the same time as introducing anti-corruption campaign, thinking we are clamping down on everything, but our model illustrates the strict rules reduce the benefit of the anti-corruption campaign. Our third theoretical result shows that fighting corruption and enhancing expertise are complements. If corruption is high, it makes sense
to constrain flies a lot, which neutralizes expertise. With low corruption and high expertise, the economy can flourish.

The paper is organized as follows. Section 2 presents a theoretical model and equilibrium of society, tigers and flies. Section 3 shows that when corruption opportunities are great, the anti-corruption campaign should crack down on both tigers and flies to benefit society. Section 4 shows strict rules and fighting corruption are substitutes. Section 5 shows fighting corruption and enhancing expertise are complements. Section 6 closes the paper and summarizes.

2 Society, Tigers and Flies: Model and Equilibrium

In this section, we present a theoretical model of an anti-corruption campaign such as the ongoing campaign in China. We abstract from the hierarchy and consider two levels: the tiger and the fly. Probably it is useful to think of these two levels as being separated by several levels in a hierarchy, so the connection between the two is not so tight. We assume that the tiger has different objectives from the society’s because of the corruption. The fly is the tiger’s subordinate and consequently his preferences are similar to the tiger’s, but he also has his own objective due to corruption. The tiger has imperfect control over the fly, executed by issuing rules that determine how much discretion the fly has. This limited control is why we think it makes sense to think of the tiger as being several levels above the fly in hierarchy that we are modelling. Flies can collect extra rents but allocating the scarce resources inefficiently.

We assume the tiger and the fly jointly make a choice represented by
a real number $X$, subject to constraints imposed by the tiger. The tiger does not have control over contracting as in the traditional agency literature following Ross (1973) and Holmstrom (1979). Instead, the tiger can only impose a constraint that the fly must choose $X$ in some interval $[\bar{X}, \bar{X}]$. In other words, the endpoints of $[X, \bar{X}]$ are chosen by the tiger.\footnote{As seems natural given the quadratic loss function and multivariate normal setting, it can be proven that if the tiger can choose any measurable subset of $\mathbb{R}$ as the restriction on $X$, the optimal choice would be a nonstochastic interval.} The tiger knows that the fly is bribed but does not have any information to condition on, so $X$ and $\bar{X}$ are constants. This limited degree of control is consistent with what is reasonable when the tiger and the fly are separated by at least several levels in the hierarchy.

The tension in the model comes from the fact that the tigers have the authority but the flies have the information. To make this simple, we will assume the flies know everything in the model, and the tigers know nothing. We can think of this as conditioning on what the tigers know and computing payoffs given what both know, without the algebraic burden of modeling this explicitly. We assume that the society’s ideal choice is $S$. The social welfare function is the expectation of

$$ W^S(X; S) = -(X - S)^2 - C_T(\gamma_T) - C_F(\gamma_F) - C_n(\sigma_n), $$

where $S \sim N(0, \sigma_S^2)$. The tiger likes to be close to society’s ideal point $S$, but can seek some private rents from deviating from society’s ideal point. The level of the anti-corruption campaign depends on two policy choices, $\gamma_T$ and $\gamma_F$. They represent the level of corruption for tigers and flies, respectively, which can be reduced by the anti-corruption campaign. $C_T(\gamma_T)$ and $C_F(\gamma_F)$ denote the costs paid by the society to reduce the corruption of tigers and flies. We assume $C_T(\gamma_T) > 0$, $C_T'(\gamma_T) < 0$, $C_T''(\gamma_T) > 0$ and $\lim_{\gamma_T \to 0} C_T(\gamma_T) = \infty$. Similarly, we assume $C_F(\gamma_F) > 0$, $C_F'(\gamma_F) < 0$, $C_F''(\gamma_F) > 0$ and $\lim_{\gamma_F \to 0} C_F(\gamma_F) = \infty$. The fly’s expertise level is inverse related with noise $\sigma_n$. $C_n(\sigma_n)$ is the cost
paid by the society to increase the fly’s expertise and decrease the noise. We assume that \( C_n(\sigma_n) > 0, C''_n(\sigma_n) < 0, C'''_n(\sigma_n) > 0 \) and \( \lim_{\sigma_n \to 0} C_n(\sigma_n) = \infty \). For simplicity, we assume the tiger and the fly do not bear any of the cost of the anti-corruption campaign, although what is important is the costs they bear are less than proportional to the benefit. Thus, tiger’s utility is

\[
(2) \quad U^T(X; S, \zeta_T) = k_T E[-(X - S)^2 + 2\zeta_T(X - S)] \\
= k_T[\gamma_T^2 + E(-(X - T)^2)]
\]

where \( \zeta_T \sim N(0, \gamma_T^2) \), drawn independently of \( S \), \( k_T > 0 \) and

\[
(3) \quad T \equiv S + \zeta_T,
\]

where \( T \sim N(0, \sigma_T^2) \) for

\[
(4) \quad \sigma_T^2 = \sigma_S^2 + \gamma_T^2
\]

because \( S \) and \( \zeta_T \) are drawn independently. We take \( k_T = 1 \), which is without loss of generality. We interpret \( T \) as the tiger’s ideal choice if the tiger knew \( S \) and \( \zeta_T \). In fact, neither \( S \) nor \( \zeta_T \) is known by the tiger, otherwise the tiger would order the fly to choose the tiger’s ideal point. The rents extracted by the tiger from the deviation from the social ideal point \( S \) are given by \( 2\zeta_T(X - S) \).

The fly does not know the tiger’s ideal choice \( T \). However, the fly knows some information \( I \), which consists of \( T \) and noise term \( \varepsilon_n \sim N(0, \sigma_n^2) \). We can write

\[
(5) \quad I = T + \varepsilon_n
\]

\footnote{Society and the tiger do not receive the same benefits from the anti-corruption campaign. However, taking \( k_T = 1 \) simplifies the algebra without affecting our results because multiplying the objective function by a constant does not change the optimal choice or ordering of alternatives.}
for $\sigma_T^2 = \sigma_T^2 + \sigma_n^2$. $\sigma_T^2 = \sigma_T^2$ when the fly has all the expertise and $\sigma_T^2 = 0$ when the fly does not have any expertise at all. As mentioned earlier, we assume for simplicity that $\varepsilon_n$ is independent of $S$ and $T$. A case can be made that $\varepsilon_n$ is correlated with $S-T$, since bribes could be for activities that produce benefits for society as well as for rent-seeking activities. We included these feature in our original analysis but assuming independence does not change the results much (Krueger, 1974).

Given information $I$, the tiger’s ideal choice $T$ is

$$T = \beta^I I + \eta_I$$

where $\beta^I = \text{cov}(T, I)/\text{var}(I) = \sigma_T^2/(\sigma_T^2 + \sigma_n^2)$, $\text{var}(\eta_I) = \sigma_T^2(1 - \beta^I)$, and $\beta^I I = E[T|I]$. We can think of $\beta^I$ as a measure of the fly’s expertise.

The fly likes the outcome to be near to the tiger’s ideal point $T$, but can get some private rents from deviating from tiger’s ideal point. The fly’s utility is expressed as

$$U^F(X; T, \zeta_F) = k_F E[-(X - T)^2 + 2\zeta_F(X - T)]$$

$$= k_F[\gamma_F^2 + E(-(X - F)^2) - \text{var}(\eta_I)]$$

where $\zeta_F \sim N(0, \gamma_F^2)$, drawn independently of $S$ and $\zeta_T$, is known by the fly. Similarly, $k_F > 0$ and we take $k_F = 1$ for simplicity. Based on (7), the fly’s ideal choice $F$ of $X$ absent constraints given $I$ can be expressed as

$$F \equiv \beta^I I + \zeta_F$$

where $F \sim N(0, \sigma_F^2)$ for

$$\sigma_F^2 = (\beta^I)^2 \sigma_T^2 + \gamma_F^2 = \beta^I \sigma_T^2 + \gamma_F^2$$

because information $I$ and $\zeta_F$ are drawn independently. The fly knows $I$ and
\(\zeta_F\), but the tiger only knows the joint distribution of these variables. The fly’s superior information limits the tiger’s ability to enforce their wishes. The rents extracted by the fly from the deviation are given by \(2\zeta_F(X - T)\). Since \(\zeta_F\) has mean zero, the tiger cannot anticipate the directions of the fly’s preferred deviation.

Given joint normality, the conditional expectations of \(T\) given \(F\) is given by a linear regression

\[
T = \beta^T F + \eta_T,
\]

where

\[
\beta^T = \frac{\text{cov}(F,T)}{\text{var}(F)} = \frac{\beta^T \sigma_T^2}{\sigma_F^2} = \frac{\beta^T (\sigma_S^2 + \gamma_T^2)}{\beta^T (\sigma_S^2 + \gamma_F^2) + \gamma_T^2} = \frac{1}{1 + \gamma_F^2 / (\beta^T \sigma_F^2)}
\]

and \(\eta_T \sim N(0, \sigma_T^2(1 - \beta^T \beta^T))\). There is no constant term in the regression because \(F\) and \(T\) both have mean zero. The regression coefficient \(\beta^T\), a number between 0 and 1, can be interpreted as the degree of alignment of the fly’s preferences with the tiger’s. We can see that the alignment is increasing in the fly’s expertise \(\gamma_F\) and decreasing in the level of the fly’s corruption opportunities \(\gamma_F\). The alignment of preferences is best when \(\beta^T = 1\) (and \(\text{var}(\eta_T) = 0\)), and worst when \(\beta^T\) approaches 0 (and \(\text{var}(\eta_T)\) approaches \(\sigma_F^2\)).

Given the restrictions from the tiger, the fly’s optimal response \(X\) is the projection of \(F\) on \([\bar{X}, \bar{X}]\), given by

\[
X = \pi(F, X, \bar{X}) = \begin{cases} 
X, & \text{if } F < X; \\
F, & \text{if } X \leq F \leq \bar{X}; \\
\bar{X}, & \text{if } \bar{X} < F. 
\end{cases}
\]
From (2) and (11) the tiger chooses $X$ and $\bar{X}$ to maximize expected utility

$$\gamma_T^2 + E[-(\pi(F, X, \bar{X}) - T)^2].$$

Also, given that $X$ and $\bar{X}$ are chosen by the tiger, social welfare is

$$E[-(\pi(F, X, X) - S)^2 - C_T(\gamma_T) - C_F(\gamma_F) - C_n(\sigma_n)].$$

The first order condition for an optimum from Equation (12), derived in Appendix B, implies that

$$\begin{cases} 
X = -\sigma_F \bar{x}(\beta^T) \\
\bar{X} = -X = \sigma_F \bar{x}(\beta^T)
\end{cases}$$

where $\bar{x}(\beta^T)$ is the solution of $\beta^T = \bar{x}N(-\bar{x})/n(\bar{x})$.

We do not know how to solve this explicitly for $\bar{x}$ given $\beta^T$, but almost just as useful, this is a parametric solution for $\beta^T$ in terms of $\bar{x}$ and $\sigma_F$. The optimal $\bar{x}$ is increasing in $\beta^T$, with $\bar{x}$ moving from 0 to $\infty$ as $\beta^T$ varies from 0 to 1, as shown in Appendix B.

## 3 Both Tigers and Flies

In this section, we fix noise $\sigma_n$ and discuss the first implications of our full model: consequences of reducing tiger’s corruption $\gamma_T$ and fly’s corruption $\gamma_F$ for society. We show that when corruption opportunities are abundant, reducing fly’s corruption ($\gamma_F$) without reducing the tiger’s corruption ($\gamma_T$) makes society worse off. In other words, when both the tiger and the fly are very corrupt, the anti-corruption campaign should get the flies and the tigers together to be effective. A second result in this section shows that tigers always prefer a relatively higher level of anti-corruption campaign than is so-
cially optimally. This is because reducing flies’ corruption opportunities $\gamma_F$ always makes tigers better off. Besides, tigers receive more than proportionally the benefits but bear less than proportionally the costs from reducing flies’ corruption.

We illustrate this section by using figures first and then state them formally in theorems. Figures 1A and 1B show how tiger’s utility (blue curves) and social welfare (black curves) vary with different levels of the corruption opportunities ($\gamma_T$ and $\gamma_F$), when noise $\sigma_n$ is fixed ($\sigma_n = 0$ and $\sigma_n = 1.5$). The contours of tiger’s utility shows that the tiger is always better off with the increases in $\gamma_T$ and decreases in $\gamma_F$. The contours of the society’s utility shows that various degrees of anti-corruption campaign leads to different impacts on the social welfare. The maximum utility that the society can achieve is denoted by the red dot. When both the tiger and the fly are very corrupt (large $\gamma_T$ and $\gamma_F$), reducing fly’s corruption ($\gamma_F$) only makes the society worse off because it aids the tiger’s corruption. When the tiger is very corrupt but the fly’s corruption opportunity is scarce ($\gamma_F \to 0$), reducing fly’s corruption ($\gamma_F$) makes the society worse off because it is more costly to the society. Formally,

**Theorem 3.1.** The anti-corruption campaign should crack down on both tigers and flies together to be effective. It is because when both the fly and the tiger are very corrupt, reducing the fly’s corruption opportunities $\gamma_F$ reduces the social welfare. We can write,

$$\frac{dW^S}{d\gamma_F} = \frac{\partial W^S}{\partial \gamma_F} |_{\bar{X}, \bar{\gamma}} + \frac{\partial W^S}{\partial \bar{X}} |_{\gamma_F} \frac{d\bar{X}}{d\gamma_F} + \frac{\partial W^S}{\partial \bar{X}} |_{\gamma_F} \frac{d\bar{X}}{d\gamma_F}^*$$

When $\gamma_F$ and $\gamma_T$ are large enough, we have $\partial W^S / \partial \gamma_F |_{\bar{X}, \bar{\gamma}} < 0$, $\partial W^S / \partial X |_{\gamma_F} \frac{dX}{d\gamma_F} = \partial W^S / \partial \bar{X} |_{\gamma_F} \frac{d\bar{X}}{d\gamma_F} > 0$, and $dW^S / d\gamma_F > 0$.

Proof: See Appendix C.
Figure 1A shows that the anti-corruption campaign should go after both the tigers and the flies to be effective. It is because when both the tiger and the fly are very corrupt (large $\gamma_T$ and $\gamma_F$), fighting fly’s corruption $\gamma_F$ only makes society worse off (black curves). Increasing tiger’s corruption opportunities $\gamma_T$ and reducing fly’s corruption opportunities $\gamma_F$ always makes the tiger better off (blue curves).

Figure 1B shows, when comparing optimal levels of anti-corruption campaign for the tigers (green solid line) and for the society (red solid line), tigers always have the tendency to choose more strict level of anti-corruption campaign than is socially optimal. It is because tigers receive higher proportion of the benefits but bear less than proportion of the costs.
We also show optimal levels of anti-corruption campaign to reduce fly’s corruption $\gamma_F$ for the tiger (green solid line) and for the society (red solid line). For any level of $\gamma_T$, the tiger always wants the fly’s corruption opportunity is zero ($\gamma_F = 0$), indicated by the horizontal line. It is because reducing $\gamma_F$ always makes the tiger better off, and the tiger gets more than proportionally the benefits but bear less than proportionally the costs. However, for each level of $\gamma_T$, the society always wants a lower level of anti-corruption campaign against the fly ($\gamma_F > 0$) because the society bears all the cost of the anti-corruption campaign. Formally,

**Theorem 3.2.** The tiger always prefers a higher level of anti-corruption campaign for the flies than is socially optimal. In particular, given $\sigma_S$, $\gamma_T$ and $\sigma_n$, the tigers want the flies to have no corruption ($\gamma_F = 0$), but society is better off with some positive level of corruption ($\gamma_F > 0$).\(^5\)

Proof: See Appendix C and D.

### 4 Information and Discretion

In this section, we fix noise $\sigma_n$, tiger’s corruption $\gamma_T$, and discuss the second implications of our model: consequences of reducing fly’s corruption $\gamma_F$ for the tiger and for the fly. We show that the anti-corruption campaign directed at the flies generate both direct and indirect benefits for the tiger. The direct benefit is that anti-corruption reform aligns the fly’s incentives closer to the tiger’s, inducing a choice closer to what the tiger wants given the same level of discretion, i.e., the same choice of $\bar{X}$ and $\bar{X}$. The indirect effect is that

\(^5\)Note: The tiger wants to reduce the fly’s corruption opportunity $\gamma_F$ to zero because the tiger does not bear any of the cost. If the tiger have a fraction of the cost, the tiger would still want less corruption for the fly than is socially optimal.
the tiger optimally takes advantage of the improved alignment of incentives by decentralizing more power and giving the fly more discretion by choosing a larger range \([X, \bar{X}]\), putting more of the decision in the hands of the agent with superior information by increasing the range of choices to the fly.

A second result in this section shows that increasing the fly’s corruption opportunities \(\gamma_F\) makes the fly better off initially when \(\gamma_F\) is small, but then worse off when \(\gamma_F\) is big. Increasing \(\gamma_F\) has two effects. It gives the fly more profits given \(X\) and \(\bar{X}\), but it also induces the tiger to shrink the interval \([X, \bar{X}]\) in anticipation of the fly deviating more from what the tiger wants, which reduces profits for the fly. When \(\gamma_F\) is small, discretion is large, the first effect dominates, and the fly profits from taking advantage of the increased corruption opportunities. When \(\gamma_F\) is large, the second effect dominates, and the fly is worse off when \(\gamma_F\) increases, because the main effect is that the tiger reduces discretion.

We illustrate this section by using figures first, and then state them formally in theorems. Figures 3A displays how the alignment of incentives \(\beta^T\) changes with the availability of relative corruption opportunities \(\gamma_F/\sigma_T\), when relative noise \(\sigma_n/\sigma_T\) is fixed at different levels. We show that the alignment of incentives \(\beta^T\) decreases in the relative corruption availability \(\gamma_F/\sigma_T\). Given the same level of relative corruption opportunities \(\gamma_F/\sigma_T\), when the fly has lower level of expertise (indicating by larger \(\sigma_n/\sigma_T\), the incentives between the tiger and the fly are worse aligned (smaller \(\beta^T\)). Given the same level of alignment \(\beta^T\) between the tiger and the fly, expertise could help fly to seek more rents from corruption opportunities (higher \(\gamma_F/\sigma_T\)).
Figure 3A shows, given $\sigma^T$, having more relative corruption opportunities (higher $\gamma_f/\sigma_f$) reduces the alignment ($\beta_T$) of the fly’s and tiger’s preferences (absent any constraint). $\gamma_f/\sigma_f$ is the scale of profitability for the relative corruption opportunities. $\sigma_n/\sigma_T$ is the level of relative noise. $\beta_T$ is the regression coefficient of tiger’s ideal point $T$ on the fly’s ideal point $F$. The tiger chooses the range of $[\bar{X}, \bar{X}]$ to constrain the fly’s choices of $X$.

Figure 3B shows, having more relative corruption opportunities (higher $\gamma_f/\sigma_f$) reduces the discretion given to the fly. The optimal interval is symmetric around $0$, i.e. $X = -\bar{X}$ in equilibrium and $\bar{X} > 0$. Larger $\gamma_f/\sigma_f$ corresponds to larger discretion given to the fly. Therefore, the larger $\gamma_f/\sigma_f$, the more the alignment between the tiger’s and fly’s incentives, and the smaller the discretion granted, i.e., the more the tiger constrains the fly.
Figure 4A shows that, given $\bar{X}$, the tiger is always better off if the corruption opportunities $\gamma_F$ for the fly are reduced because this aligns their incentives better. The tiger can further exploit the improved alignment of incentives by giving the fly more discretion. When $\gamma_F$ is small, large discretion to the fly makes the tiger better off because more jobs are delegated to the agent with better information. When $\gamma_F$ is large, large discretion to the fly makes the tiger worse off because the fly cares more about the rents from bribery than the tiger’s objective.

Figure 4B shows that the anti-corruption campaign makes the fly better off when $\gamma_F$ is small and worse off when $\gamma_F$ is big. When $\gamma_F$ is small, the anti-corruption campaign makes the fly worse off because benefits from corruption opportunities are larger than the increases in the discretion. When $\gamma_F$ is large, the anti-corruption campaign makes the fly better off because the increase in the discretion is more important than the rents collected from the corruption opportunities.
Figure 3B displays how the amount of relative discretion $\bar{X}/\sigma_T$ changes with the availability of relative corruption opportunities $\gamma_F/\sigma_T$, when relative noise $\sigma_n/\sigma_T$ is fixed at different levels. We show that the fly’s relative discretion $\bar{X}/\sigma_T$ decreases with the relative corruption availability $\gamma_F/\sigma_T$. When the relative corruption availability $\gamma_F/\sigma_T$ is small, better aligning the interests for the fly and the tiger (increasing $\beta^T$ in Figure 3A), so the tiger gives more discretion to the fly (increasing $\bar{X}/\sigma_T$). When the relative corruption opportunity $\gamma_F/\sigma_T$ is large, anticipating that the fly will collect more rents and care less about the tiger’s ideal choice (decreasing $\beta^T$ in Figure 3A), the tiger gives the fly very small range of $X$ to choose (decreasing $\bar{X}/\sigma_T$). Given the same level of relative corruption opportunities $(\gamma_F/\sigma_T)$, the tiger would like to give more relative discretion $\bar{X}/\sigma_T$ to the fly when the fly is more skillful (smaller $\sigma_n/\sigma_T$). Given the same level of relative discretion $\bar{X}/\sigma_T$, expertise could help fly to collect more rents from corruption opportunities (larger $\gamma_F/\sigma_T$).

Figure 4A shows the change in the tiger’s utility $U^T$ with the change in the corruption opportunities $\gamma_F$ at different levels of discretion $\bar{X}$, when noise $\sigma_n$ is fixed at 1. Reducing fly’s corruption $\gamma_F$ always makes the tiger better off and the tiger can achieve the highest utility at the optimal level of $\bar{X}$. When fly is very corrupt (large $\gamma_F$), large discretion ($\bar{X}$) to the fly makes the tiger worse off. It is because within this range, the fly cares more about the rents collected from bribery than the tiger’s preference. When fly’s corruption opportunities are scarce (small $\gamma_F$), large discretion ($\bar{X}$) to the fly makes the tiger better off due to the benefits from delegating the jobs to the agent with better decision making. Formally,

**Theorem 4.1.** Reducing the fly’s corruption opportunities $\gamma_F$ always makes the tiger better off because the reduction aligns the fly’s incentives better with the tiger’s. Reducing the fly’s corruption opportunities $\gamma_F$ also generates indirect benefits to the tiger because the tiger optimally gives the fly more discretion so that more decisions are made by the agent with superior infor-
We have

\( \frac{dU}{d\gamma} = \frac{\partial U}{\partial \gamma} |_{\bar{X}, \bar{X}} + \frac{\partial U}{\partial \bar{X}} |_{\gamma} \star \frac{d\bar{X}}{d\gamma} + \frac{\partial U}{\partial \bar{X}} |_{\gamma} \star \frac{d\bar{X}}{d\gamma} \)

(1) \( \frac{dU}{d\gamma} |_{\bar{X}, \bar{X}} < 0 \) (direct benefit)

which says that given the level of discretion, reducing \( \gamma \) makes the tiger better off.

(2) \( \frac{d\bar{X}}{d\gamma} > 0 \) and \( \frac{d\bar{X}}{d\gamma} < 0 \) (indirect benefit)

which says that the tiger prefers to give the fly more discretion as \( \gamma \) decreases.

Proof: See Appendix D.

Figure 4B shows how the fly’s utility \( U \) varies with the availability of corruption \( \gamma \) at different levels of discretion \( \bar{X} \), when noise \( \sigma_n \) is fixed at 1. When \( \bar{X} \) is fixed, given the same availability of the corruption \( \gamma \), the fly is always better off with more discretion. Besides, the fly’s utility with the optimal \( \bar{X} \) indicates that anti-corruption campaign makes the fly sometimes worse off and sometimes better off. For instance, when \( \gamma \) is small, anti-corruption makes the fly worse off. Because in this range, the reduction in corruption opportunities is more important than the increase in discretion, which is already large. When \( \gamma \) is large, anti-corruption makes the fly better off. Because in this range, the additional discretion is more important than the reduction in corruption opportunities. Formally, we have

**Theorem 4.2.** Reducing the fly’s corruption opportunities \( \gamma \) makes the fly worse off initially when \( \gamma \) is small and then better off when \( \gamma \) is big. We can write

\( \frac{dU}{d\gamma} = \frac{\partial U}{\partial \gamma} |_{\bar{X}, \bar{X}} + \frac{\partial U}{\partial \bar{X}} |_{\gamma} \star \frac{d\bar{X}}{d\gamma} + \frac{\partial U}{\partial \bar{X}} |_{\gamma} \star \frac{d\bar{X}}{d\gamma} \)

We have
(1) $\frac{\partial U^F}{\partial \gamma_F}|_{\bar{X}, \bar{X}} > 0$, but $\frac{\partial U^F}{\partial \bar{X}}|_{\gamma_F} \cdot \frac{d\bar{X}}{d\gamma_F} = \frac{\partial U^F}{\partial \bar{X}}|_{\gamma_F} \cdot \frac{d\bar{X}}{d\gamma_F} < 0$, which does not allow us to sign $\frac{dU^F}{d\gamma_F}$.
(2) $\frac{dU^F}{d\gamma_F} > 0$ when $\gamma_F$ is close enough to 0, and
(3) $\frac{dU^F}{d\gamma_F} < 0$ when $\gamma_F$ is large enough.

Proof: See Appendix D.

5 Expertise

In this section, we discuss the last implications from our model: consequences of reducing noise $\sigma_n$ for the tiger and the society. We show that fighting corruption and enhancing expertise are complements. From the tiger’s perspective, if corruption is high, $\bar{X}$ and $\bar{X}$ will be close to zero, and expertise will not matter much. Strict rules tend to neutralize expertise. If expertise is low, the benefit of discretion is small no matter how low corruption is, and corruption will not matter much. Fighting flies’ corruption probably will not be worth the cost. In addition, both the direct and indirect benefits of reducing corruption are small. Conversely, if corruption is small and expertise is high, the range of $[X, \bar{X}]$, will be chosen to be large and the expertise will have a big impact on the choice. The results for society are similar, but if the tiger’s corruption is large enough both expertise and anti-corruption for flies may benefit the tiger but not society.
Figures 5A and 5B show how the social welfare and the tiger’s utility change with noise $\sigma_n$ when corruption opportunities are scarce and are very large, respectively. We show that the tiger is always better off if the fly’s expertise is increased (through training to reduce $\sigma_n$) because this aligns their incentives better. The tiger can further exploit the improved alignment of incentives by giving the fly more discretion.

In Figure 5A, when $\gamma_F$ is small, large discretion to the fly makes the tiger better off because more jobs are delegated to the agent with better information. Prior the cost, the interests of society and the tiger are well aligned. It is still true when the benefits of training fly are more than the cost. Thus, with more discretion and higher expertise, the economy can flourish. In Figure 5B, when $\gamma_F$ is big, the tiger gives the fly very small range of $[\bar{X}, \tilde{X}]$. Fly’s expertise does not matter much because strict rules neutralize expertise. In addition, increasing fly’s expertise makes the society worse off because it aids the tiger’s corruption.
We illustrate this section by using figures first, and then state them formally in theorems. Figures 5A and 5B show how the tiger’s utility and social welfare (with and without cost) change with noise $\sigma_n$ when corruption opportunities are small and very large, respectively. We show that tiger’s utility is always decreasing with the increase with $\sigma_n$. In Figure 5A, when the corruption opportunities are scarce, the interests of society are well aligned with the interests of tiger’s prior the cost. Reducing the noise makes the society better off when the corruption is low. After the cost, the society still has the same interests as the tiger has when the benefits of training the fly is more than the costs. In Figure 5B, when both tigers and flies are very corrupt, social welfare is increasing with the noise $\sigma_n$ with and without cost. It is because that the purpose of training is to help fly perform better for what the tiger wants. When the tiger is very corrupt, increasing fly’s expertise aids the tiger’s corruption, and makes the society worse off. Formally, we have

**Theorem 5.1.** Increasing the fly’s expertise (through training that reduces $\sigma_n$), always makes the tiger better off because the expertise aligns the fly’s incentives better with the tiger’s. Increasing the fly’s expertise also generates indirect benefits to the tiger because the tiger optimally gives the fly more discretion so that more decisions are made by the agent with superior information. We can write

$$\frac{dU_T}{d\sigma_n} = \frac{\partial U_T}{\partial \sigma_n}|_{\bar{X}, \bar{X}} + \frac{\partial U_T}{\partial \bar{X}}|_{\gamma_F, \beta_I} * \frac{d\bar{X}}{d\sigma_n} + \frac{\partial U_T}{\partial \bar{X}}|_{\gamma_F, \beta_I} * \frac{d\bar{X}}{d\sigma_n}$$

We have

(1) $\frac{\partial U_T}{\partial \sigma_n}|_{\bar{X}, \bar{X}} < 0$ (direct benefit)

which says that given the level of discretion, reducing $\sigma_n$ makes the tiger better off.

(2) $d\bar{X}/d\sigma_n > 0$ and $d\bar{X}/d\sigma_n < 0$ (indirect benefit)

which says that the tiger prefers to give the fly more discretion as $\sigma_n$ decreases.
Proof: See Appendix E.

**Theorem 5.2.** The anti-corruption campaign should be accompanied by training flies to have more expertise. With low corruption and high expertise, the economy can flourish. We can write

\[
\frac{dW^S}{d\sigma_n} = \frac{\partial W^S}{\partial \sigma_n}_{|X,\bar{X}} + \frac{\partial W^S}{\partial \bar{X}}_{|\bar{t}, \beta} * \frac{dX}{d\sigma_n} + \frac{\partial W^S}{\partial X} \bar{t} * \frac{d\bar{X}}{d\sigma_n}
\]

When \(\gamma_F\) and \(\gamma_T\) are small, we have \(\partial W^S/\partial \sigma_n|_{X,\bar{X}} < 0, \partial W^S/\partial X|_{\gamma_F,\beta} * dX/d\sigma_n = \partial W^S/\partial \bar{X}|_{\gamma_F,\beta} * d\bar{X}/d\sigma_n > 0\), and \(dW^S/d\sigma_n < 0\). When \(\gamma_F\) and \(\gamma_T\) are very large, we have \(\partial W^S/\partial \sigma_n|_{X,\bar{X}} < 0, \partial W^S/\partial X|_{\gamma_F,\beta} * dX/d\sigma_n = \partial W^S/\partial \bar{X}|_{\gamma_F,\beta} * d\bar{X}/d\sigma_n > 0\), and \(dW^S/d\sigma_n > 0\).

Proof: See Appendix E.

When the fly is very corrupt but the tiger is not, training the fly does not have a significant impact on the society, because when the fly is very corrupt, tiger will choose very small range of \([X, \bar{X}]\). Strict rules neutralize expertise. However, when the tiger is very corrupt but the fly is not, training the fly will cause a loss to the society, because when the fly’s corruption opportunities are scarce, tiger will give more discretion to the fly. This induces the fly to do more what the corrupt tiger wants, and makes the society worse off.

### 6 Conclusion

This paper builds a theoretical model of an anti-corruption campaign like the ongoing campaign in China. The issues of balancing control with provision of incentives is more universal and arises in all governments. We focus on the true anti-corruption campaign and discuss effects of reductions in corruption opportunities to society, tigers and flies. We have three main conclusions.
First, fighting the flies without fighting the tigers reduces welfare. If the tigers are very corrupt, reducing flies’ corruption helps the tigers to implement their corruption. Second, strict rules and fighting corruption are substitutes. To take full advantage of the aligned incentives, a reduction of corruption should be accompanied by more discretion. Third, fighting corruption and enhancing expertise are complements. If corruption is high, discretion will be close to zero, and expertise will not matter much. Strict rules tend to neutralize expertise. If expertise is low, the benefit of discretion is small no matter how low corruption is, and corruption will not matter much. If expertise is low, both the direct and indirect benefits of reducing corruption are small. Conversely, if corruption is small and expertise is high, discretion will be chosen to be large and the expertise will have a big impact on the choice. An effective fight against corruption at all levels can make society more efficient by aligning incentives and allowing the allocation of control rights to the people with the information needed to make decisions. The greatest improvement will occur if fighting of corruption is accompanied by efficient delegation of decision-making and the development of the appropriate level of expertise.

Appendix A  Facts about Normal Distribution

Lemma A1. Let $n(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2}$ and $N(x) = \int_{y=-\infty}^{x} n(y)\, dy$ be the unit normal density and cumulative distribution functions, respectively. Thus,

1. $N'(x) = n(x)$, $n'(x) = -xn(x)$, $n''(x) = -n(x) + x^2n(x)$;
2. $N(x)$ is an increasing 1-1 function mapping $(-\infty, +\infty)$ onto $(0,1)$;
3. For $x < 0$, $g(x) \equiv N(x) + n(x)x + N(x)x^2$ is an increasing 1-1 function mapping $(-\infty, 0)$ to $(0, +\infty)$;
4. For $x < 0$, $p(x) \equiv g(x)x^2/N(x)$ is an increasing 1-1 function mapping $(-\infty, 0)$ to $(0,1)$;
5. For $x > 0$, $f(x) \equiv xN(-x)/n(x)$ is a monotonically increasing 1-1 function mapping $(0, +\infty)$ onto $(0,1)$. 

22
Proof of Lemma A1(3).

\[
\frac{dg(x)}{dx} = \left[ n(x) - n(x) - \frac{n(x)}{x^2} + \frac{n(x)}{x^2} - \frac{2N(x)}{x^3} \right] = -\frac{2N(x)}{x^3}
\]

Therefore, \(g(x)\) is increasing with \(x\) when \(x < 0\). As \(x \to -\infty\), \(g(x) \to 0\) because each term goes to 0; As \(x \to 0\), \(g(x) \to +\infty\). Thus, for \(x \in (-\infty, 0)\), \(g(x) \in (0, +\infty)\). ■

Proof of Lemma A1(4).

\[
g(x) = \frac{N(y)}{y^2} \bigg|_{y=-\infty}^{x} - \int_{y=-\infty}^{x} \frac{n(y)}{y^2} \, dy = \frac{N(x)}{x^2} - \int_{y=-\infty}^{x} \frac{n(y)}{y^2} \, dy
\]

\[
p(x) = \left( \frac{N(x)}{x^2} - \int_{y=-\infty}^{x} \frac{n(y)}{y^2} \, dy \right) \frac{x^2}{N(x)} = 1 - \frac{1}{N(x)} \int_{y=-\infty}^{x} \frac{x^2n(y)}{y^2} \, dy
\]

Then

\[
0 < \frac{1}{N(x)} \int_{y=-\infty}^{x} \frac{x^2n(y)}{y^2} \, dy = 1 - p(x) < \frac{1}{N(x)} \int_{y=-\infty}^{x} \frac{y^2n(y)}{y^2} \, dy = 1
\]

Thus, for \(x \in (-\infty, 0)\), \(p(x) \in (0, 1)\). ■

Proof of Lemma A1 (5).

\[
\frac{df(x)}{dx} = \frac{d}{dx} \left( \frac{xN(-x)}{n(x)} \right)
\]

\[
= \frac{N(-x)}{n(x)} - x + \frac{x^2N(-x)}{n(x)}
\]

\[
= \frac{x^2}{n(x)} \left( \frac{N(-x) + n(-x)/(-x) + N(-x)/(-x)^2}{g(-x) > 0} \right) > 0
\]

Thus, \(f(x)\) is strictly increasing with \(x\).

When \(x \to 0\), \(f(x) \to 0\). When \(x \to +\infty\), \(N(-x) \to 0\), \(n(x)/x \to 0\),
Thus, for $x > 0$, $f(x) \equiv xN(-x)/n(x)$ is a monotonically increasing 1-1 function mapping $(0, +\infty)$ onto $(0,1)$.

### Appendix B  Characterizing optimal $x$ and $\bar{x}$

Since $T$ and $F$ are jointly normal with 0 mean, we can write $T = \beta^T F + \eta_T$, where $\eta_T$ is independent of $F$, $\beta^T = \text{cov}(T,F)/\text{var}(F) = \beta^T \sigma_T^2/\sigma_F^2$, and $\sigma_T^2 = \beta^T \beta T \sigma_T^2 + \text{var}(\eta_T)$. Therefore, $\text{var}(\eta_T) = \sigma_T^2(1 - \beta^T \beta T)$.

Tiger’s utility is

\begin{equation}
E[U_T] = \gamma_T^2 + E[-(X - T)^2]
\end{equation}

or alternatively

\begin{equation}
E[(X - T)^2] = E[(X - (\beta^T F + \eta_T))^2]
= E[(X - \beta^T F)^2 - 2\eta_T(X - \beta^T F) + \eta_T^2]
= E[(X - \beta^T F)^2] + \text{var}(\eta_T)
\end{equation}

$E[\eta_T F] = 0$ by the property of regression (10), and $E[\eta_T X] = 0$ because $E[\eta_T] = 0$ and the fly’s choice of $X$ depends on realized $X$ but not realized
\( \eta_T \) (or equivalently depends on \( F \) but not \( T \)). From (11), (15) and (16),

\[
E[U_T] = \gamma_T^2 - E[(\pi(F, X, \bar{X}) - \beta^T F)^2] - \text{var}(\eta_T)
\]

\[
= \gamma_T^2 - \frac{1}{\sqrt{2\pi\sigma_F}} \int_{F=-\infty}^{X} (X - \beta^T F)^2 e^{-\frac{F^2}{2\sigma_F^2}} dF
\]

\[
- \frac{1}{\sqrt{2\pi\sigma_F}} \int_{F=X}^{\infty} (F - \beta^T F)^2 e^{-\frac{F^2}{2\sigma_F^2}} dF
\]

\[
- \frac{1}{\sqrt{2\pi\sigma_F}} \int_{F=\bar{X}}^{\infty} (\bar{X} - \beta^T F)^2 e^{-\frac{F^2}{2\sigma_F^2}} dF - \text{var}(\eta_T)
\]

Let \( \varphi \equiv F/\sigma_F \sim N(0, 1) \) and \( \bar{x} = \bar{X}/\sigma_F \). Thus, the tiger’s first-order condition for \( \bar{X} \) of the above is

\[
0 = \frac{\partial}{\partial \bar{X}} E[(-\pi(F, X, \bar{X}) - \beta^T F)^2]
\]

\[
= -\frac{1}{\sqrt{2\pi\sigma_F}} \int_{F=\bar{X}}^{+\infty} 2(\bar{X} - \beta^T F)e^{-\frac{F^2}{2\sigma_F^2}} dF
\]

\[
= -2\sigma_F \int_{\varphi=\bar{x}}^{+\infty} (\bar{x} - \beta^T \varphi) \frac{1}{\sqrt{2\pi}} e^{-\frac{\varphi^2}{2}} d\varphi
\]

\[
= -2\sigma_F \left[ \bar{x} \int_{\varphi=\bar{x}}^{+\infty} n(\varphi) d\varphi - \beta^T \int_{\varphi=\bar{x}}^{+\infty} \varphi n(\varphi) d\varphi \right]
\]

\[
= -2\sigma_F \left[ \bar{x} N(\bar{x}) - \beta^T n(\bar{x}) \right]
\]

\[
= -2\sigma_F \bar{x} \left[ \bar{x} N(\bar{x}) - \beta^T n(\bar{x}) \right]
\]

Similarly, the tiger’s first-order condition for \( X \) is

\[
0 = \frac{\partial}{\partial X} E[(-\pi(F, X, \bar{X}) - \beta^T F)^2]
\]

\[
= -2\sigma_F n(x) \left[ \frac{\bar{x} N(\bar{x})}{n(\bar{x})} + \beta^T \right]
\]

Since the positive coefficient \( \sigma_F \) does not affect Equation (18), the optimal \( \bar{x} \) is independent of \( \sigma_F \) (given \( \beta^T \)), i.e. \( \bar{X} = \sigma_F \bar{x} \) where \( \bar{x} \) is a constant solving
Equation (18) given $\beta^T$. Thus,

$$
\beta^T = \frac{\bar{x}N(-\bar{x})}{n(\bar{x})} = \frac{-xN(x)}{n(x)}
$$

Thus, Lemma A1(5) can be used to show that, 1) the optimal choice of $\bar{x}$ and $\bar{x}$ is determined uniquely by the first order condition (18), and 2) the optimal $\bar{x}$ (respectively $x = -\bar{x}$) is increasing (respectively decreasing) in $\beta^T$. From Lemma A1(5), Equation (18) has a unique solution given $\beta^T$, call it $\bar{x}(\beta^T)$. Furthermore, by Equation (18) and Lemma A1(5), $dU^T/d\bar{X} > 0$ when $\bar{x} < \bar{x}(\beta^T)$ and $dU^T/d\bar{X} < 0$ when $\bar{x} > \bar{x}(\beta^T)$. It implies that $\bar{x}(\beta^T)$ (or more precisely $\bar{X} = \sigma_F \bar{x}(\beta^T)$) is the tiger’s unique optimal choice. This proof is similar that $\bar{x}(\beta^T)$ is also optimal. Furthermore, Equation (18) and Lemma A1(5) imply that $\bar{x}(\beta^T)$ is an increasing function.

**Appendix C   Both Tigers and Flies**

**C.1 Social Welfare and $\gamma_F$**

**Proof of Theorem 3.1.** Since S and T are jointly normal with 0 mean, we can write

$$
S = \beta^S F + \eta_S,
$$

where $\eta_S$ is independent of F. $\beta^S = cov(S, F)/\text{var}(F) = \beta^T \sigma_S^2/\sigma_F^2$, and $\sigma_S^2 = \beta^T \beta^S \sigma_S^2 + \text{var}(\eta_S)$. Therefore, $\text{var}(\eta_S) = \sigma_S^2 (1 - \beta^T \beta^S)$.

Social welfare is

$$
E[W^S] = E[-(X - S)^2 - C_T(\gamma_T) - C_F(\gamma_F) - C_n(\sigma_n)]
$$
or alternatively

\[ E[(X - S)^2] = E[X - (\beta S F + \eta S)^2] \]
\[ = E[(X - \beta S F)^2 - 2\eta S(X - \beta S F) + \eta S^2] \]
\[ = E[(X - \beta S F)^2] + \text{var}(\eta_S) \]

\( E[\eta_S X] = 0 \) because \( E[\eta_S] = 0 \) and the fly's choice of \( X \) depends on realized \( X \) but not realized \( \eta_S \) (or equivalently depends on \( F \) but not \( S \)). \( E[\eta_S F] = 0 \) by the property of regression (21). From (11), (22) and (23),

\[ E[W^S] = E[-(\pi(S_F, X, \tilde{X}) - \beta S F)] - \text{var}(\eta_S) - C_T(\gamma_T) - C_F(\gamma_F) - C_n(\sigma_n) \]
\[ = \int_{\varphi=-\infty}^{\infty} \left(-X^2 + 2X \frac{\beta F \sigma_S^2}{\sigma_F} \varphi - \frac{\beta F^2 \sigma_s^4}{\sigma_F^2} \varphi^2 \right) n(\varphi) d\varphi + \int_{\varphi=X/\sigma_F}^{\infty} \left(-\frac{\sigma_F^2 \varphi^2 + 2\beta F \sigma_S^2 \varphi^2}{\sigma_F^2} - \frac{\beta F^2 \sigma_s^4 \varphi^2}{\sigma_F^2} \right) n(\varphi) d\varphi \]
\[ + \int_{\varphi=X/\sigma_F}^{\infty} \left(-\frac{\sigma_F^2 - C_T(\gamma_T) - C_F(\gamma_F) - C_n(\sigma_n) \right) \]
\[ = \int_{\varphi=-\infty}^{\infty} \left(-X^2 + 2X \frac{\beta F \sigma_S^2}{\sigma_F} \varphi - \frac{\beta F^2 \sigma_s^4}{\sigma_F^2} \varphi^2 \right) n(\varphi) d\varphi + \int_{\varphi=X/\sigma_F}^{\infty} \left(-\frac{\sigma_F^2 \varphi^2 + 2\beta F \sigma_S^2 \varphi^2}{\sigma_F^2} - \frac{\beta F^2 \sigma_s^4 \varphi^2}{\sigma_F^2} \right) n(\varphi) d\varphi \]
\[ + \int_{\varphi=\tilde{X}/\sigma_F}^{\infty} \left(-\tilde{X}^2 + 2\tilde{X} \frac{\beta F \sigma_S^2}{\sigma_F} \varphi - \frac{\beta F^2 \sigma_s^4 \varphi^2}{\sigma_F^2} \right) n(\varphi) d\varphi \]
\[ + \int_{\varphi=\tilde{X}/\sigma_F}^{\infty} \left(-\tilde{X}^2 + 2\tilde{X} \frac{\beta F \sigma_S^2}{\sigma_F} \varphi - \frac{\beta F^2 \sigma_s^4 \varphi^2}{\sigma_F^2} \right) n(\varphi) d\varphi \]
\[ + \int_{\varphi=\tilde{X}/\sigma_F}^{\infty} \left(-\tilde{X}^2 + 2\tilde{X} \frac{\beta F \sigma_S^2}{\sigma_F} \varphi - \frac{\beta F^2 \sigma_s^4 \varphi^2}{\sigma_F^2} \right) n(\varphi) d\varphi \]

We have \( E[W^S] \) depends on \( X, \tilde{X}, \sigma_F, [\beta F], [\sigma_S] \), where

(1) \( \sigma_F \) depends on \( [\sigma_T], [\sigma_n], \gamma_F \);

\[ \footnote{We use \( [\ ] \) to present this parameter is fixed as a constant in the proof through this section.} \]
(2) $X, \bar{X}$ depend on $\sigma_F, \beta^T$;
(3) $\beta^T$ depends on $[\sigma_n], [\sigma_T]$ and $\gamma_F$.

Then,
\[
\frac{dE[W^S]}{d\gamma_F} = \frac{\partial E[W^S]}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial E[W^S]}{\partial C_F(\gamma_F)} \frac{dC_F(\gamma_F)}{d\gamma_F} + \frac{\partial E[W^S]}{\partial \bar{X}} \frac{d\bar{X}}{d\gamma_F} \frac{\partial \bar{X}}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial E[W^S]}{\partial \bar{X}} \frac{d\bar{X}}{d\gamma_F} \frac{\partial \bar{X}}{\partial \beta^T} \frac{d\beta^T}{d\sigma_F} \frac{d\sigma_F}{d\gamma_F}.
\]

I. Fixed $X$ and $\bar{X}$

For now, we are fixing $\sigma_T^2, \sigma_n^2, \sigma_S^2, \gamma_T, X$ and $\bar{X}$ throughout this section. Since $\sigma_F^2 = (\sigma_T^2)^2/\sigma_T^2 + \gamma_F^2 = (\sigma_T^2)^2/(\sigma_T^2 + \sigma_n^2) + \gamma_F^2$, then
\[
\frac{d\sigma_F}{d\gamma_F} = \frac{d}{d\gamma_F} \left( \left( \frac{\sigma_T^2}{\sigma_T^2 + \sigma_n^2} + \gamma_F^2 \right)^{\frac{1}{2}} \right) = \frac{\gamma_F}{\sigma_F} > 0
\]
From equations (24) and (25),

\begin{equation}
\frac{\partial E[W^S]}{\partial \sigma_F} = \int_{\varphi=-\infty}^{\varphi=\sigma_F} \frac{2X \beta' \sigma^2}{\sigma^2_F} \varphi n(\varphi) d\varphi + \int_{\varphi=\sigma_F}^{\varphi=\infty} -2\sigma_F \varphi^2 n(\varphi) d\varphi
\end{equation}

\begin{equation}
+ \int_{\varphi=\bar{\sigma}_F}^{\varphi=\infty} -2\frac{X \beta' \sigma^2}{\sigma^2_F} \varphi n(\varphi) d\varphi
\end{equation}

\begin{equation}
= 2X \beta^2 n(\varphi)|_{-\infty}^{\varphi=\sigma_F} -2\sigma_F(-\varphi n(\varphi))|_{\sigma_F}^{\infty} -2\sigma_F N(\varphi)|_{\sigma_F}^{\infty} + 2X \beta^2 n(\varphi)|_{\sigma_F}^{\infty}
\end{equation}

\begin{equation}
= 2X \beta^2 n(X/\sigma_F) + 2\sigma_F \frac{\bar{X}}{\sigma_F} n(\bar{X}/\sigma_F) - 2\sigma_F \frac{\bar{X}}{\sigma_F} n(X/\sigma_F)
\end{equation}

\begin{equation}
- 2\sigma_F [N(\bar{X}/\sigma_F) - N(X/\sigma_F)] - 2X \beta^2 n(\bar{X}/\sigma_F)
\end{equation}

\begin{equation}
= 2\sigma_F(\beta^2 - 1)[X/\sigma_F n(X/\sigma_F) - \bar{X}/\sigma_F n(\bar{X}/\sigma_F)] - 2\sigma_F [N(\bar{X}/\sigma_F) - N(X/\sigma_F)]
\end{equation}

\begin{equation}
= 2\sigma_F(\beta^2 - 1)[\bar{x} n(\bar{x}) - \bar{n}(\bar{x})] - 2\sigma_F [N(\bar{x}) - N(\bar{x})]
\end{equation}

\begin{equation}
= 4\sigma_F(1 - \beta^2) \bar{x} n(\bar{x}) - 2\sigma_F [2N(\bar{x}) - 1]
\end{equation}

Let \( k^S(\bar{x}) \equiv 4\sigma_F(1 - \beta^2) \bar{x} n(\bar{x}) - 2\sigma_F [2N(\bar{x}) - 1] \), when \( \bar{x} > 0 \)

\begin{equation}
k^S(\bar{x}) = 2\sigma_F [2(1 - \beta^2)(n(\bar{x}) - \bar{x}^2 n(\bar{x})) - 2n(\bar{x})]
\end{equation}

\begin{equation}
= 2\sigma_F [2n(\bar{x}) - 2\bar{x}^2 n(\bar{x}) - 2\beta^2 n(\bar{x}) + 2\beta^2 \bar{x}^2 n(\bar{x}) - 2n(\bar{x})]
\end{equation}

\begin{equation}
= 2\sigma_F [-2\beta^2 n(\bar{x}) + 2(\beta^2 - 1)\bar{x}^2 n(\bar{x})] < 0
\end{equation}

Thus, \( k^S(\bar{x}) \) is decreasing with \( \bar{x} \) when \( \bar{x} > 0 \). When \( \bar{x} \to 0 \), \( k^S(\bar{x}) \to 0 \).

Thus, for \( 0 < \bar{x} < +\infty \), \( k^S(\bar{x}) < 0 \) implies that \( k^S(\bar{x}) < 0 \).

When \( \gamma_F \) is very large, \( \bar{X} \to 0 \), \( C_F(\gamma_F) \to 0 \), \( C'_F(\gamma_F) \to 0 \), \( \partial E[W^S]/\partial \sigma_F < 0 \). Therefore,

\begin{equation}
\frac{\partial E[W^S]}{\partial \gamma_F} \bigg|_{X,\bar{X}} = \frac{\partial E[W^S]}{\partial \sigma_F} \cdot \frac{d\sigma_F}{d\gamma_F} + \frac{\partial E[W^S]}{\partial C_F(\gamma_F)} \cdot \frac{dC_F(\gamma_F)}{d\gamma_F}
\end{equation}

\begin{equation}
= 2\gamma_F(\beta^2 - 1)[x n(x) - \bar{x} n(\bar{x})] - 2\gamma_F [N(\bar{x}) - N(x)] - C'_F(\gamma_F) < 0
\end{equation}

Thus, when both \( X \) and \( \bar{X} \) are fixed, reducing the fly’s corruption opportunities (\( \gamma_F \)) makes the society better off when \( \gamma_F \) is very large.
II. Tiger Chooses $X$ and $\bar{X}$ Optimally in Response to $\gamma_F$

Now, we consider what happens to social welfare with changes in $\gamma_F$ when the tiger chooses $\bar{X}$ and $\bar{X}$ optimally in response to $\gamma_F$.

\[
\frac{\partial E[W_S]}{\partial \bar{X}} = \int_{X/\sigma_F}^{+\infty} (-2\bar{X} + 2\frac{\beta^I\sigma^2_\theta}{\sigma_F} n(\varphi) d\varphi
\]

\[
= -2\bar{X} N(-\bar{X}/\sigma_F) + 2\frac{\beta^I\sigma^2_\theta}{\sigma_F} n(\bar{X}/\sigma_F)
\]

\[
= -2\sigma_F n(\bar{X}/\sigma_F) \left[ \frac{(\bar{X}/\sigma_F) N(-\bar{X}/\sigma_F)}{n(X/\sigma_F)} - \frac{\beta^I\sigma^2_\theta}{\sigma_F^2} \right]
\]

\[
= -2\frac{\beta^I\gamma_T^2 n(\bar{X}/\sigma_F)}{\sigma_F} - 2\frac{\beta^I\gamma_T^2 n(\bar{X}/\sigma_F)}{\sigma_F}
\]

From equation (20)

\[
\frac{\partial \beta^T}{\partial \bar{X}} \bigg|_{\sigma_F} = \frac{\partial}{\partial \bar{X}} \left\{ \frac{(\bar{X}/\sigma_F) N(-\bar{X}/\sigma_F)}{n(X/\sigma_F)} \right\}
\]

\[
= \frac{N(-\bar{X}/\sigma_F)}{\sigma_F n(\bar{X}/\sigma_F)} + \frac{-\bar{X} n(\bar{X}/\sigma_F)}{\sigma_F^2 n(\bar{X}/\sigma_F)} + \frac{\bar{X}^2 N(-\bar{X}/\sigma_F) n(\bar{X}/\sigma_F)}{\sigma_F^3 n^2(\bar{X}/\sigma_F)}
\]

\[
= \frac{1}{\sigma_F n(\bar{x})} \left[ N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x}) \right]
\]

Thus,

\[
\frac{\partial \bar{X}}{\partial \beta^T} \bigg|_{\sigma_F} = \frac{\sigma_F n(\bar{x})}{N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})}
\]

When $\beta^T$ is fixed, $\bar{X}/\sigma_F$ is also fixed. Thus, $\partial(\bar{X}/\sigma_F) = 0$, $\partial \log(\bar{X}/\sigma_F) = \partial \log(\bar{X}) - \partial \log(\sigma_F) = 0$, $\partial \bar{X}/\bar{X} = \partial \sigma_F/\sigma_F$, and

\[
\frac{\partial \bar{X}}{\partial \sigma_F} \bigg|_{\beta^T} = \frac{\bar{X}}{\sigma_F} = \bar{x}
\]
\[
\begin{align*}
(32) \quad & \frac{d \beta^T}{d \gamma_F} = \frac{d}{d \gamma_F} \left( \sigma_T^4 + \sigma_T^2 \gamma_F^2 + \sigma_n^2 \gamma_F^2 \right) \\
& = - \frac{2 \gamma_F \sigma_T^4 \left( \sigma_T^2 + \sigma_n^2 \right)}{[(\sigma_T^2 + \sigma_n^2) \left( \frac{\sigma_T^4}{\sigma_T^2 + \sigma_n^2} + \gamma_F^2 \right)]^2} \\
& = - \frac{2 \gamma_F \sigma_T^4}{(\sigma_T^2 + \sigma_n^2) \sigma_T^4} = - \frac{2 \gamma_F \beta^T}{\sigma_T^4}
\end{align*}
\]

\[
\begin{align*}
(33) \quad & \frac{\partial E[W^S]}{\partial X} \cdot \left( \frac{\partial X}{\partial \sigma_F} * \frac{d \sigma_F}{d \gamma_F} + \frac{\partial X}{\partial \beta^T} * \frac{d \beta^T}{d \gamma_F} \right) \\
& = \frac{2 \beta^T \gamma_F^2 n(\bar{x})}{\sigma_F} \left[ \frac{\bar{x} \cdot \sigma_F}{\sigma_F} + \frac{\sigma_F n(\bar{x})}{\bar{N}(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \cdot \left( \frac{-2 \gamma_F \beta^T}{\sigma_F^2} \right) \right] \\
& = \frac{2 \beta^T \gamma_F^2 n(\bar{x})}{\sigma_F^2} \left[ \frac{\bar{x} - \frac{2 \beta^T n(\bar{x})}{\bar{N}(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})}}{\bar{N}(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x}) - \bar{x}} \right]
\end{align*}
\]

Similarly,

\[
\begin{align*}
(34) \quad & \frac{\partial E[W^S]}{\partial X} \cdot \left( \frac{\partial X}{\partial \sigma_F} * \frac{d \sigma_F}{d \gamma_F} + \frac{\partial X}{\partial \beta^T} * \frac{d \beta^T}{d \gamma_F} \right) \\
& = \frac{\partial E[W^S]}{\partial X} \cdot \left( \frac{\partial X}{\partial \sigma_F} * \frac{d \sigma_F}{d \gamma_F} + \frac{\partial X}{\partial \beta^T} * \frac{d \beta^T}{d \gamma_F} \right) \\
& = \frac{2 \beta^T \gamma_F^2 n(\bar{x})}{\sigma_F^2} \left[ \frac{2 \beta^T n(\bar{x})}{\bar{N}(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x}) - \bar{x}} \right]
\end{align*}
\]
From Equations (28), (33) and (34)

\[
\frac{dE[W_S]}{d\gamma_F} = 2\gamma_F(\beta^S - 1)[xn(\bar{x}) - \bar{x}n(\bar{x})] - 2\gamma_F[N(\bar{x}) - N(\bar{x})] - C'_F(\gamma_F) \\
+ 2 \left( \frac{2\beta^T \gamma_F \gamma^2_T n(\bar{x})}{\sigma^2_F} \right) \cdot \left[ \frac{2\beta^T n(\bar{x})}{N(\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x}) - \bar{x}} \right] \\
= 4\gamma_F(1 - \beta^S) \cdot \bar{x}n(\bar{x}) - 2\gamma_F[N(\bar{x}) - N(\bar{x})] - C'(\gamma_F) \\
+ 4 \left( \frac{\beta^T \gamma^2_T n(\bar{x})}{\sigma^2_F} \right) \cdot \left[ \frac{2\beta^T n(\bar{x})}{N(\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x}) - \bar{x}} \right] \\
= 4\gamma_F \bar{x} \left[ (1 - \beta^S)n(\bar{x}) - \frac{N(\bar{x}) - N(\bar{x})}{2\bar{x}} \right] \\
+ \frac{\beta^T \gamma^2_T n(\bar{x})}{\sigma^2_F} \cdot \left( \frac{2\bar{x}N(-\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x}) - \bar{x}} \right) - C'_F(\gamma_F) \\
= 4\gamma_F \bar{x} \left[ (1 - \beta^S)n(\bar{x}) - \frac{N(\bar{x}) - N(\bar{x})}{2\bar{x}} \right] \\
+ \frac{\beta^T \gamma^2_T n(\bar{x})}{\sigma^2_F} \cdot \left( \frac{2}{1 - \bar{x}n(\bar{x})/N(-\bar{x}) + \bar{x}^2} - 1 \right) - C'_F(\gamma_F) \\
\]

When both $\gamma_T$ and $\gamma_F$ are very large, $\bar{x} \to 0$, $\beta^S \to 0$, $C_F(\gamma_F) \to 0, C'_F(\gamma_F) \to 0$. Thus, Equation (35) can be written as

\[
\frac{dE[W_S]}{d\gamma_F} \approx \frac{4\gamma_F \bar{x} \left[ (1 - \beta^S)n(0) - n(0) + \frac{\beta^T \gamma^2_T n(0)}{\sigma^2_F} \right] \cdot (2 + 4n(0)\bar{x} - 1)} - C'_F(\gamma_F) \\
\approx \frac{4\gamma_F \bar{x} \left[ - \beta^S n(0) + \frac{\beta^T \gamma^2_T n(0)}{\sigma^2_F} \right] \cdot (1 + 4n(0)\bar{x})}{C'_F(\gamma_F)} \\
\approx \frac{4\gamma_F \bar{x} \left[ - \beta^T \sigma^2_S n(0) + \frac{\beta^T \gamma^2_T n(0)}{\sigma^2_F} \right] + \frac{4\beta^T \gamma^2_T n(0)^2}{\sigma^2_F} - \bar{x}}{C'_F(\gamma_F)} \\
\approx \frac{4\beta^T \gamma_F \bar{x}n(0)}{\sigma^2_F} \left( \gamma^2_T - \sigma^2_S \right) \cdot C'_F(\gamma_F) \to 0 \\
> 0 \text{ when } \gamma_T \text{ is large enough} \\
\]

When both $\gamma_F$ and $\gamma_F$ are large enough, social welfare is increasing with $\gamma_F$. Thus, when both the fly and the tiger are very corrupt, reducing fly’s corruption opportunities $\gamma_F$ without reducing tiger’s
corruption opportunities $\gamma_T$ makes the society worse off.

\[
\frac{dE[W_S]}{d\gamma_F} \approx 4\gamma_F \bar{x} \left( 1 - \beta_S n(0) - n(0) + \frac{\beta^I \gamma^2 T n(0)}{\sigma^2_F} \right) \ast (2 + 4n(0)\bar{x} - 1) - C'_F(\gamma_F)
\]

\[
\approx 4\gamma_F \bar{x} \left[ -\beta_S n(0) + \frac{\beta^I \gamma^2 T n(0)}{\sigma^2_F} \ast (1 + 4n(0)\bar{x}) \right] - C'_F(\gamma_F)
\]

\[
\approx 4\gamma_F \bar{x} \left[ -\beta^I \sigma^2_S n(0) + \frac{\beta^I \gamma^2 T n(0)}{\sigma^2_F} + \frac{4\beta^I \gamma^2 T n^2(0)\bar{x}}{\sigma^2_F} \right] - C'_F(\gamma_F)
\]

\[
\approx \frac{4\beta^I \gamma_F \bar{x} n(0) (\gamma^2_T - \sigma^2_S)}{\sigma^2_F} - C'_F(\gamma_F) > 0
\]

>0 when $\gamma_T$ is large enough

\[
>0
\]
Appendix D  Information and Discretion

D.1 Tiger and $\gamma_F$

Proof of Theorem 4.1. Based on equation (12), the tiger’s utility is

(36) $E[U_T] = \gamma_T^2 + E[-(\pi(\sigma_F \varphi, X, \bar{X}) - \beta^T F)^2] - \text{var}(\eta_T)$

$= \gamma_T^2 + E[-(\pi(\sigma_F \varphi, X, \bar{X}) - \beta^T \sigma_F \varphi)^2] - \text{var}(\eta_T)$

$= \gamma_T^2 + \int_{\varphi=-\infty}^{X/\sigma_F} -(X - \beta^T \sigma_F \varphi)^2 n(\varphi) \, d\varphi + \int_{\varphi=\bar{X}/\sigma_F}^{\infty} -(\sigma_F \varphi + \beta^T \sigma_F \varphi)^2 n(\varphi) \, d\varphi$

$+ \int_{\varphi=\bar{X}/\sigma_F}^{\infty} -(\bar{X} + \beta^T \sigma_F \varphi)^2 n(\varphi) \, d\varphi - \text{var}(\eta_T)$

$= \gamma_T^2 + \int_{\varphi=-\infty}^{X/\sigma_F} (-X^2 + 2X \frac{\beta^I \sigma_T^2 \varphi}{\sigma_F} - \frac{(\beta^I)^2 \sigma_T^4}{\sigma_F^2} \varphi^2) n(\varphi) \, d\varphi$

$+ \int_{\varphi=\bar{X}/\sigma_F}^{\infty} (-X^2 + 2X \frac{\beta^I \sigma_T^2 \varphi}{\sigma_F} - \frac{(\beta^I)^2 \sigma_T^4}{\sigma_F^2} \varphi^2) n(\varphi) \, d\varphi - \sigma_T^2 + \frac{(\beta^I)^2 \sigma_T^4}{\sigma_F^2}$

$= \gamma_T^2 + \int_{\varphi=-\infty}^{X/\sigma_F} (-X^2 + 2X \frac{\beta^I \sigma_T^2 \varphi}{\sigma_F} \varphi) n(\varphi) \, d\varphi + \int_{\varphi=\bar{X}/\sigma_F}^{\infty} (-\sigma_T^2 \varphi^2 + 2 \beta^I \sigma_T^2 \varphi^2) n(\varphi) \, d\varphi$

$+ \int_{\varphi=\bar{X}/\sigma_F}^{\infty} (-\bar{X}^2 + 2 \bar{X} \frac{\beta^I \sigma_T^2 \varphi}{\sigma_F} \varphi) n(\varphi) \, d\varphi - \sigma_T^2$

$E[U_T]$ depends on $\sigma_F$, $[\sigma_T]$, $[\beta^I]$, where $\sigma_F$ depends on $[\sigma_T]$, $\gamma_F$, $[\sigma_n]$.

(37) $\frac{d\sigma_F}{d\gamma_F} = \frac{\gamma_F}{\sigma_F}$
Thus, for 0 < \bar{x} < +\infty, k_T(\bar{x}) < 0 implies that k_T(\bar{x}) < 0.

(39) \frac{\partial E[U^T]}{\partial \gamma_F} |_{X, \bar{X}} = \frac{\partial E[U^T]}{\partial \sigma_F} \ast \frac{d\sigma_F}{d\gamma_F} < 0

For fixed X and \bar{X}, tiger’s utility is decreasing with \gamma_F. Also, this is true if the tiger chooses X and \bar{X}, since the tiger will choose X and \bar{X} to maximize utility and the pointwise maximum of decreasing functions is decreasing.
From Equations (30), (31), (32),

\[ \frac{d\bar{X}}{d\gamma_F} = \frac{\partial \bar{X}}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial \bar{X}}{\partial \beta^T} \frac{d\beta^T}{d\gamma_F} \]

\[ = \bar{x} \frac{\gamma_F}{\sigma_F} + \frac{\sigma_F n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \left( - \frac{2\gamma_F \beta^T}{\sigma_F^2} \right) \]

\[ = \gamma_F \bar{x} \frac{1}{\sigma_F} \left( 1 - \frac{2N(-\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \right) \]

\[ = \gamma_F \bar{x} \frac{1}{\sigma_F} \left( 1 - \frac{2}{p(-\bar{x})} \right) < 0 \]

The final inequality follows from Lemma A1(4), which defines \( p(x) \) and implies \( p(x) \in (0, 1) \), so \( 2/p(x) \in (2, +\infty) \). Similarly,

\[ \frac{dX}{d\gamma_F} = -\frac{d\bar{X}}{d\gamma_F} > 0 \]

Thus, discretion is decreasing with fly’s corruption opportunities \( \gamma_F \).

\[ \square \]
D.2 Fly and $\gamma_F$

Proof of Theorem 4.2. Based on (7), the fly’s utility is

$$E[U_F] = \gamma_F^2 + E[-(\pi(\sigma_F\varphi, X, \bar{X}) - \sigma_F\varphi)^2] - \text{var}(\eta_I)$$

$$= \gamma_F^2 + \int_{\varphi=-\infty}^{X/\sigma_F} -(X - \sigma_F\varphi)^2 n(\varphi) d\varphi + \int_{\varphi=\infty}^{+\infty} -(\bar{X} - \sigma_F\varphi)^2 n(\varphi) d\varphi - \sigma_T^2(1 - \beta^I)$$

$$= \gamma_F^2 + \int_{\varphi=-\infty}^{X/\sigma_F} (-X^2 + 2X\sigma_F\varphi - \sigma_F^2\varphi^2) n(\varphi) d\varphi$$

$$+ \int_{\varphi=\infty}^{+\infty} (-\bar{X}^2 + 2\bar{X}\sigma_F\varphi - \sigma_F^2\varphi^2) n(\varphi) d\varphi - \sigma_T^2(1 - \beta^I)$$

$E[U_F]$ depends on $\sigma_F$, $[\beta^I]$, $X$, $\bar{X}$, where

1. $\sigma_F$ depends on $[\sigma_T]$, $[\sigma_n]$, $\gamma_F$;
2. $X$, $\bar{X}$ depend on $\sigma_F$, $\beta^T$;
3. $\beta^T$ depends on $\gamma_F$, $[\sigma_T]$ and $[\sigma_n]$.

Thus,

$$\frac{dE[U_F]}{d\gamma_F} = \frac{\partial E[U_F]}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial E[U_F]}{\partial X} \left( \frac{\partial X}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial X}{\partial \beta^T} \frac{d\beta^T}{d\gamma_F} \right)$$

$$+ \frac{\partial E[U_F]}{\partial \bar{X}} \left( \frac{\partial \bar{X}}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial \bar{X}}{\partial \beta^T} \frac{d\beta^T}{d\gamma_F} \right)$$

I. Fixed $X$ and $\bar{X}$

For now, we are fixing $\sigma_T$, $X$ and $\bar{X}$ throughout this section. From Equations
(42),

\[
\frac{\partial E[U_F]}{\partial \sigma_F} |_{X, \bar{X}} = 2\gamma_F + \sigma_F - \frac{X}{\sigma_F}(-X^2 + 2X\sigma_F - \sigma_F^2 \bar{X}^2 n(X/\sigma_F))
\]

\[
+ \int \phi = -\infty \left(2X \varphi - 2\sigma_F \varphi^2 \right) n(\varphi) d\varphi
\]

\[
+ \left(\frac{X}{\sigma_F} \right)^2 \left(-X^2 + 2X\sigma_F - \sigma_F^2 \bar{X}^2 \right) n(\bar{X}/\sigma_F)
\]

\[
+ \int \phi = \infty \left(2X \varphi - 2\sigma_F \varphi^2 \right) n(\varphi) d\varphi
\]

\[
= 2\sigma_F + \int \phi = -\infty \left(2X \varphi - 2\sigma_F \varphi^2 \right) n(\varphi) d\varphi + \int \phi = \infty \left(2X \varphi - 2\sigma_F \varphi^2 \right) n(\varphi) d\varphi
\]

\[
= 2\sigma_F - 2X n(X/\sigma_F) + 2\sigma_F n(X/\sigma_F - 2\sigma_F \bar{X}^2 n(X/\sigma_F) + 2X n(X/\sigma_F - 2\sigma_F n(X/\sigma_F - 2\sigma_F (1 - N(X/\sigma_F)) = 2\sigma_F \bar{X}^2 n(X/\sigma_F) - 2\sigma_F N(X/\sigma_F) = 2\sigma_F [N(\bar{x}) - N(x)] > 0
\]

From Equations (26), (43), and (44),

\[
\frac{\partial E[U_F]}{\partial \gamma_F} |_{X, \bar{X}} = \frac{\partial E[U_F]}{\partial \sigma_F} |_{X, \bar{X}} \sigma_F \gamma_F
\]

\[
= 2\sigma_F [N(\bar{x}) - N(x)] \cdot \gamma_F = 2\sigma_F [N(\bar{x}) - N(x)] \cdot \gamma_F > 0
\]

Therefore, when \(X\) and \(\bar{X}\) are fixed, the fly's utility is increasing with \(\gamma_F\).

**II. Tiger Chooses \(X\) and \(\bar{X}\) Optimally**

Now consider what happens when the tiger chooses \(X\) and \(\bar{X}\) optimally in
response to $\gamma_F$,

\begin{equation}
\left. \frac{\partial E[U_F]}{\partial \bar{X}} \right|_{\sigma_F} = -\frac{1}{\sigma_F}(-\bar{X}^2 + 2\bar{X}^2 - X^2)n(\bar{X}/\sigma_F) + \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} (-2\bar{X} + 2\varphi \sigma_F)n(\varphi) \, d\varphi
\end{equation}

\begin{align*}
&= -2\bar{X}N(\varphi)\bigg|_{\bar{X}/\sigma_F}^{+\infty} - 2\sigma_F n(\varphi)\bigg|_{\bar{X}/\sigma_F}^{+\infty} \\
&= -2\bar{X}N(-\bar{X}/\sigma_F) + 2\sigma_F n(\bar{X}/\sigma_F) \\
&= -2\sigma_F n(\bar{X}/\sigma_F)\left(\frac{\bar{X}/\sigma_F N(-\bar{X}/\sigma_F)}{n(\bar{X}/\sigma_F)} - 1\right) \\
&= -2\sigma_F n(\bar{X}/\sigma_F)(\beta^T - 1) = \frac{2\gamma_F^2 n(\bar{x})}{\sigma_F}
\end{align*}

\begin{equation}
\left. \frac{\partial E[U_F]}{\partial \bar{X}} \right|_{\sigma_F} \ast \left( \frac{\partial \bar{X}}{\partial \sigma_F} \ast \frac{d\sigma_F}{d\gamma_F} + \frac{\partial \bar{X}}{\partial \beta^T} \ast \frac{d\beta^T}{d\gamma_F} \right)
\end{equation}

\begin{align*}
&= 2\gamma_F^2 n(\bar{x}) \ast \left[ \bar{x} \ast \frac{\gamma_F F}{\sigma_F} + \frac{\sigma_F n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \ast \left( -\frac{2\gamma_F^2 n(\bar{x})}{\sigma_F^2} \right) \right] \\
&= 2\gamma_F^2 n(\bar{x}) \left[ \bar{x} - \frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \right] \\
&= -2\gamma_F^2 n(\bar{x}) \left[ \frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} - \bar{x} \right]
\end{align*}

Similarly,

\begin{equation}
\left. \frac{\partial E[U_F]}{\partial \bar{X}} \right|_{\sigma_F} \ast \left( \frac{\partial \bar{X}}{\partial \sigma_F} \ast \frac{d\sigma_F}{d\gamma_F} + \frac{\partial \bar{X}}{\partial \beta^T} \ast \frac{d\beta^T}{d\gamma_F} \right)
\end{equation}

\begin{align*}
&= \left. \frac{\partial E[U_F]}{\partial \bar{X}} \right|_{\sigma_F} \ast \left( \frac{\partial \bar{X}}{\partial \sigma_F} \ast \frac{d\sigma_F}{d\gamma_F} + \frac{\partial \bar{X}}{\partial \beta^T} \ast \frac{d\beta^T}{d\gamma_F} \right) \\
&= -2\gamma_F^2 n(\bar{x}) \left[ \frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} - \bar{x} \right]
\end{align*}

From Equations (45), (47), and (48), Equation (43) can be expressed as

\begin{equation}
\frac{dE[U_F]}{d\gamma_F} = 2\gamma_F [N(\bar{x}) - N(x)] - 2 \frac{2\gamma_F^2 n(\bar{x})}{\sigma_F} \left[ \frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} - \bar{x} \right]
\end{equation}
When $\gamma_F$ is very large, $X \to 0$, $\bar{X} \to 0$ and $\beta^T \to 0$. Since $f'(x) \equiv \lim_{\Delta \to 0} \frac{f(x+\Delta)-f(x)}{\Delta}$, and $f'(x) \equiv \lim_{\Delta \to 0} \frac{f(x+\Delta)-f(x-\Delta)}{2\Delta}$, we have

$$\frac{N(\bar{x})-N(x)}{2\bar{x}} \to n(0) \tag{50}$$

$$\frac{\gamma_F^2}{\sigma_F^2} = 1 - \beta^T = 1 - \frac{(\bar{X}/\sigma_F)N(-\bar{X}/\sigma_F)}{n(\bar{X}/\sigma_F)} \approx 1 - \frac{(\bar{X}/\sigma_F)(1/2 - n(0)\bar{X}/\sigma_F)}{n(0)}$$

$$= 1 - \frac{1}{2n(0)} \left( \frac{\bar{X}}{\sigma_F} \right) + \left( \frac{\bar{X}}{\sigma_F} \right)^2 \approx 1 - \frac{1}{2n(0)} \left( \frac{\bar{X}}{\sigma_F} \right) = 1 - \frac{\bar{x}}{2n(0)} \tag{51}$$
Thus, equation (49) can be written as

\[
\frac{dE[U_F]}{d\gamma_F} \approx 4\bar{x}^2 \gamma_F \left[ \frac{n(0)}{\bar{x}} - \gamma_F \frac{n(0)}{\sigma_F^2} \frac{2(2 + 2n(0)\bar{x} - 1)}{1 - \bar{x}n(0)} \right]
\]

Therefore, the fly’s utility is decreasing with \( \gamma_F \) when \( \gamma_F \) is large.

When \( \gamma_F \) is small, \( X \to -\infty, \bar{X} \to +\infty \) and \( \beta^T \to 1 \). Thus, Equation
can be written as
\[
\frac{dE[U_F]}{d\gamma_F} \approx 4\bar{x}^2 \gamma_F \left[ \frac{N(\bar{x}) - N(-\bar{x})}{2\bar{x}^2} - \frac{\gamma_F^2 n(\bar{x})}{\sigma_F^2} \left( \frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x}) - \bar{x}} \right) \right]
\]
\[
= 4\bar{x}^2 \gamma_F \left[ \frac{N(\bar{x}) - N(-\bar{x})}{2\bar{x}^2} - \frac{\gamma_F^2 n(\bar{x})}{\sigma_F^2} \left( \frac{2}{1 - \bar{x}^2/\beta^T + \bar{x}^2 - 1} \right) \right]
\]
\[
\approx 4\bar{x}^2 \gamma_F \left[ \frac{N(\bar{x}) - N(-\bar{x})}{2\bar{x}^2} - \frac{\gamma_F^2 n(\bar{x})}{\sigma_F^2} \left( \frac{2}{\bar{x}^2/\beta^T + \bar{x}^2 - 1} \right) \right]
\]
\[
\approx 2\gamma_F > 0
\]

The fly’s utility is increasing with \( \gamma_F \) when \( \gamma_F \) is close to 0. ■

Appendix E  Expertise

E.1  Tiger’s utility and \( \sigma_n \)

Proof of Theorem 5.1. Now, we consider what happens to the tiger in response to fly’s expertise \( \sigma_n \). From (36), we have \( E[U^T] \) depends on \( \sigma_F \), \( \beta^I \), \( [\sigma_T] \), where
(1) \( \sigma_F \) depends on \( [\sigma_T] \), \([\gamma_F]\), \( \sigma_n \);
(2) \( \beta^I \) depends on \( [\sigma_T] \), \( \sigma_n \).
For fixed $\bar{X}$ and $\bar{x}$, tiger’s utility is decreasing with $\sigma_n$. Also, this is true if the tiger chooses $\bar{X}$ and $\bar{x}$, since the tiger will choose $\bar{X}$ and
\(X\) to maximize utility and the pointwise maximum of decreasing functions is decreasing.

From Equations (30), (31), (32),

\[
\frac{d\bar{X}}{d\sigma_n} = \frac{\partial \bar{X}}{\partial \sigma_F} \ast \frac{d\sigma_F}{d\sigma_n} + \frac{\partial \bar{X}}{\partial \beta^T} \ast \frac{d\beta^T}{d\sigma_n}
\]

\[
= \bar{x} \ast \left( -\frac{(\beta^I)^2 \sigma_n}{\sigma_F} \right) + \frac{\sigma_F n(\bar{x})}{N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \ast \left( -\frac{2(\beta^I)^2 \sigma_n \gamma^2_F}{\sigma^4_F} \right)
\]

\[
= \left( -\frac{(\beta^I)^2 \sigma_n}{\sigma_F} \right) \ast \left( \bar{x} + \frac{2\gamma^2_F \ast n(\bar{x})/\bar{x}^2}{\sigma_F^2 \ast \left[ N(-\bar{x}) + n(\bar{x})/(\bar{x}) + N(-\bar{x})/(-\bar{x})^2 \right]} \right) < 0
\]

The final inequality follows from Lemma A1(3), which defines \(g(x)\) and implies \(g(x) \in (0, +\infty)\). Similarly,

\[
\frac{d\bar{X}}{d\sigma_n} = -\frac{d\bar{X}}{d\sigma_n} > 0
\]

Thus, discretion is decreasing with noise \(\sigma_n\).

\[
\frac{d^2 E[U^T]}{d\gamma_F d\sigma_n} = \frac{d\sigma_F}{d\gamma_F} \left( \frac{\partial^2 U^T}{\partial \sigma_F^2} + \frac{\partial^2 U^T}{\partial \sigma_F \partial \beta^T} \ast \frac{d\beta^T}{d\sigma_n} \right)
\]

\textbf{E.2 Social welfare and }\sigma_n\

\textbf{Proof of Theorem 5.2.} Next, we consider what happens to the society in response to fly’s expertise \(\sigma_n\). From (24), we have \(E[W^S]\) depends on \(X, \hat{X}, \sigma_F, \beta^I, [\sigma_S]\), where

1. \(\sigma_F\) depends on \([\sigma_T], [\gamma_F], \sigma_n\),
2. \(\beta^I\) depends on \([\sigma_T], \sigma_n\),
3. \(X, \hat{X}\) depend on \(\sigma_F, \beta^T\),
4. \(\beta^T\) depends on \([\gamma_F], [\sigma_T]\) and \(\sigma_n\).
\[
\frac{dE[W^S]}{d\sigma_n} = \frac{\partial E[W^S]}{\partial \sigma_T} \frac{d\sigma_T}{d\sigma_n} + \frac{\partial E[W^S]}{\partial \sigma_F} \frac{d\sigma_F}{d\sigma_n} + \frac{\partial E[W^S]}{\partial \beta^I} \frac{d\beta^I}{d\sigma_n} + \frac{\partial E[W^S]}{\partial C_n(\sigma_n)} \frac{dC_n(\sigma_n)}{d\sigma_n}
\]

I. Fixed \(X\) and \(\bar{X}\)

Now, let’s fixed \(\sigma_T^2, \sigma_F^2, \gamma_F, \gamma_T, X\) and \(\bar{X}\). Since \(\sigma_T^2 = (\sigma_T^2)^2 / \sigma_T^2 + \gamma_T^2 = \sigma_T^2 / (\sigma_T^2 + \sigma_F^2) + \gamma_T^2\), then

\[
\frac{d\sigma_T}{d\sigma_n} = \frac{d}{d\sigma_n} \left( \frac{\sigma_T^2}{\sigma_T^2 + \sigma_F^2} \right) = \frac{1}{2\sigma_T^4} \left( -\frac{\sigma_T^2 \sigma_F^2}{(\sigma_T^2 + \sigma_F^2)^2} \right)
= -\frac{\sigma_T^2 \sigma_F}{\sigma_T^4} < 0
\]

\[
\frac{d\beta^I}{d\sigma_n} = \frac{d}{d\sigma_n} \left( \frac{\sigma_T^2}{\sigma_T^2 + \sigma_F^2} \right) = -\frac{2\sigma_T^2 \sigma_F}{(\sigma_T^2 + \sigma_F^2)^2} = -\frac{2\sigma_T^2 \sigma_F}{\sigma_T^4}
\]

\[
\frac{\partial E[W^S]}{\partial \beta^I} = \int_{\varphi = -\infty}^{X/\sigma_F} 2X \frac{\sigma_T^2}{\sigma_F} \varphi_n(\varphi) d\varphi + \int_{\varphi = X/\sigma_F}^{X/\sigma_F} 2\sigma_T^2 \phi^2 n(\varphi) d\varphi + \int_{\varphi = X/\sigma_F}^{+\infty} 2\sigma_T^2 \phi n(\varphi) d\varphi

= -2X \frac{\sigma_T^2}{\sigma_F} n(\varphi) \bigg|_{-\infty}^{X/\sigma_F} + 2\sigma_T^2 \int_{\varphi = X/\sigma_F}^{X/\sigma_F} [n''(\varphi) + n(\varphi)] d\varphi - 2X \frac{\sigma_T^2}{\sigma_F} n(\varphi) \bigg|_{X/\sigma_F}^{+\infty}

= -2X \frac{\sigma_T^2}{\sigma_F} n(X/\sigma_F) + 2\sigma_T^2 \left[ -\varphi n(\varphi) \bigg|_{X/\sigma_F}^{X/\sigma_F} + N(\varphi) \bigg|_{X/\sigma_F}^{X/\sigma_F} \right] + 2X \frac{\sigma_T^2}{\sigma_F} n(X/\sigma_F)

= -2\sigma_T^2 n(\bar{x}) - 2\sigma_T^2 \bar{x} n(\bar{x}) + 2\sigma_T^2 [N(\bar{x}) - N(\bar{x})] + 2\bar{x} \sigma_T^2 n(\bar{x})

= 2\sigma_T^2 [N(\bar{x}) - N(\bar{x})]
\]
Therefore, from Equations (??), (26) and (??),

\[
\frac{\partial E[W_S]}{\partial \sigma_n} \bigg|_{X, \bar{X}} \\
= \frac{\partial E[W_S]}{\partial \sigma_F} \frac{d \sigma_F}{d \sigma_n} + \frac{\partial E[W_S]}{\partial \sigma_I} \frac{d \sigma_I}{d \sigma_n} + \frac{\partial E[W_S]}{\partial C_n} \frac{d C_n}{d \sigma_n} \\
= \left(2\sigma_F (\beta^S - 1)[\bar{\sigma}(\bar{X}) - \bar{\sigma}(\bar{X})] - 2\sigma_F [N(\bar{X}) - N(\bar{X})]\right) * \left(-\frac{\sigma_n \sigma_T^4}{\sigma_F \sigma_I^4}\right) \\
+ 2\sigma_S^2 [N(\bar{X}) - N(\bar{X})] * \left(-\frac{2\sigma_T^2 \sigma_n}{\sigma_I^4}\right) - C'_n(\sigma_n) \\
= \frac{4\sigma_n \sigma_T^4}{\sigma_I^4} (\beta^S - 1)\bar{\sigma}(\bar{X}) + \frac{2\sigma_n \sigma_T^4}{\sigma_I^4} [N(\bar{X}) - N(\bar{X})] - \frac{4\sigma_n \sigma_T^2 \sigma_F^2}{\sigma_I^4} [N(\bar{X}) - N(\bar{X})] - C'_n(\sigma_n) \\
= \frac{4\sigma_n \sigma_T^4}{\sigma_I^4} (\beta^S - 1)\bar{\sigma}(\bar{X}) + \frac{2\sigma_n \sigma_T^2 (\sigma_F^2 - \sigma_S^2)}{\sigma_I^4} [N(\bar{X}) - N(\bar{X})] - C'_n(\sigma_n)
\]

**Case one:**

When both \( \gamma_F \) and \( \gamma_T \) are small, \( X \to -\infty, \bar{X} \to +\infty \).

\[
\frac{\partial E[W_S]}{\partial \sigma_n} \bigg|_{X, \bar{X}} \approx \frac{4\sigma_n \sigma_T^4}{\sigma_I^4} (\beta^S - 1)\bar{\sigma}(\bar{X}) + \frac{2\sigma_n \sigma_T^2 (\sigma_F^2 - \sigma_S^2)}{\sigma_I^4} [N(+\infty) - N(-\infty)] - C'_n(\sigma_n) \\
\approx \frac{4\sigma_n \sigma_T^4}{\sigma_I^4} (\beta^S - 1)\bar{\sigma}(\bar{X}) + \frac{2\sigma_n \sigma_T^2 (\gamma^2 - \sigma_S^2)}{\sigma_I^4} - C'_n(\sigma_n)
\]

Thus, for fixed \( X \) and \( \bar{X} \), when both tiger and fly are not very corrupt, society is worse off with the increase in the noise \( \sigma_n \) before cost.

**Case two:**
When both $\gamma_F$ and $\gamma_T$ are very large, $X \to 0$, $\bar{X} \to 0$.

$$\frac{\partial E[W_S]}{\partial \sigma_n} \bigg|_{\bar{X}, \bar{X}} \approx 4\frac{\sigma_n\sigma_T^2}{\sigma_I^4}\left[\sigma_T^2(\beta^S - 1)n(0) + (\sigma_T^2 - 2\sigma_S^2)n(0)\right] - C_n'(\sigma_n)$$

$$= 4\frac{\sigma_n\sigma_T^2}{\sigma_I^4}\left[(\sigma_T^2\beta^S - \sigma_T^2 + \sigma_T^2 - 2\sigma_S^2)n(0)\right] - C_n'(\sigma_n)$$

$$= 4\frac{\sigma_n\sigma_T^2}{\sigma_I^4}\left[(\sigma_T^2\beta^S - \sigma_T^2 + \sigma_T^2 - 2\sigma_S^2)n(0)\right] - C_n'(\sigma_n)$$

$$= 4\frac{\sigma_n\sigma_T^2}{\sigma_I^4}\left[(\sigma_S^2\beta^I\sigma_T^2 - 2\sigma_S^2)n(0)\right] - C_n'(\sigma_n)$$

$$= 4\frac{\sigma_n\sigma_T^2}{\sigma_I^4}\left[(\sigma_S^2(\beta^T - 2)n(0)\right] - C_n'(\sigma_n) < 0$$

Thus, for fixed $X$ and $\bar{X}$, when both tiger and fly are very corrupt, society is worse off with the increase in the noise $\sigma_n$ before cost.

**II. Tiger chooses $X$ and $\bar{X}$ optimally in response with $\gamma_F$**

Next, we consider what happens to the social welfare with changes in $\sigma_n$ when the tiger chooses $X$ and $\bar{X}$ in response to $\gamma_F$.

$$\frac{d\beta^T}{d\sigma_n} = \frac{d}{d\sigma_n}\left(\frac{\sigma_T^4}{\sigma_T^4 + \sigma_T^2\gamma_F^2 + \sigma_n^2\gamma_F^2}\right) = -\frac{2\sigma_T^4\sigma_n\gamma^2}{(\sigma_T^4 + \sigma_T^2\gamma_F^2 + \sigma_n^2\gamma_F^2)^2}$$

$$= -\frac{2\sigma_T^4\sigma_n\gamma^2}{\sigma_T^4 + \sigma_T^2\gamma_F^2 + \sigma_n^2\gamma_F^2} = -\frac{2\sigma_T^4\sigma_n\gamma^2 \gamma_F^2}{\sigma_T^4 \sigma_n^2 \gamma_F^2}$$

$$= -\frac{2(\beta^I)^2\sigma_n\gamma_F^2}{\sigma_T^4 \sigma_n^2 \gamma_F^2}$$

47
\[ (64) \quad \frac{\partial E[W^S]}{\partial X} = 2 \left( \beta I \right)^3 \gamma_T^2 \sigma_n(\bar{x}) \left( \frac{2\gamma_T^2 n(\bar{x})}{\sigma_F^2} \right) \left[ \bar{x} + \frac{2\gamma_T n(\bar{x})}{\sigma_F^2} \right] \] 

Similarly,

\[ (65) \quad \frac{\partial E[W^S]}{\partial X} = 2 \left( \beta I \right)^3 \gamma_T^2 \sigma_n(\bar{x}) \left( \frac{2\gamma_T^2 n(\bar{x})}{\sigma_F^2} \right) \left[ \bar{x} + \frac{2\gamma_T n(\bar{x})}{\sigma_F^2} \right] \] 

**Case one:** When both \( \gamma_F \) and \( \gamma_T \) are small, \( X \to -\infty, \bar{X} \to +\infty \).
Equation (57) can be expressed as:

\[
\frac{d E[W^S]}{d \sigma_n} \approx 4\sigma_n \sigma_T^3 (\beta^S - 1) \bar{x} n(\bar{x}) + \frac{2\sigma_n \sigma_T^2 (\gamma_T^2 - \bar{\sigma}_S^2)}{\sigma_T^4} \left[ N(\bar{x}) - N(x) \right] - C'_n(\sigma_n)
\]

\[
+ \frac{4(\beta^T)^3 \gamma_T^2 \sigma_T n(\bar{x})}{\sigma_T^2} \left[ \bar{x} + \frac{2\gamma_T^2 n(\bar{x})}{\sigma_T^2} \cdot \left( N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x}) \right) \right]
\]

\[
\approx \frac{4\sigma_n \sigma_T^2 \bar{x} n(\bar{x})}{\sigma_T^4} \left[ (\beta^S - 1) \bar{x} n(\bar{x}) + (\sigma_T^2 - 2\bar{\sigma}_S^2) \cdot \frac{N(\bar{x}) - N(x)}{2\bar{x}} \right]
\]

\[
+ \frac{\sigma_T^4 \gamma_T^2 n(\bar{x})}{\sigma_T^4} \left( 1 + \frac{2\gamma_T^2 n(\bar{x})}{\sigma_T^2} \cdot [N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})] \right) - C'_n(\sigma_n)
\]

\[
\approx \frac{4\sigma_n \sigma_T^2 \bar{x} n(\bar{x})}{\sigma_T^4} \left[ (\beta^S - 1) \bar{x} n(\bar{x}) + (\sigma_T^2 - 2\bar{\sigma}_S^2) \cdot \frac{N(\bar{x}) - N(x)}{2\bar{x}} \right]
\]

\[
+ \frac{\sigma_T^4 \gamma_T^2 n(\bar{x})}{\sigma_T^4} \left( 1 + \frac{2\gamma_T^2 n(\bar{x})}{\sigma_T^2} \cdot [N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})] \right) - C'_n(\sigma_n)
\]

\[
\approx \frac{4\sigma_n \sigma_T^2 \bar{x} n(\bar{x})}{\sigma_T^4} \left[ (\beta^S - 1) \bar{x} n(\bar{x}) + (\sigma_T^2 - 2\bar{\sigma}_S^2) \cdot \frac{N(\bar{x}) - N(x)}{2\bar{x}} \right]
\]

\[
+ \frac{\sigma_T^4 \gamma_T^2 n(\bar{x})}{\sigma_T^4} \left( 1 + \frac{2\gamma_T^2 n(\bar{x})}{\sigma_T^2} \cdot [N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})] \right) - C'_n(\sigma_n)
\]

\[
\approx \frac{4\sigma_n \sigma_T^2 \bar{x} n(\bar{x})}{\sigma_T^4} \left[ (\beta^S - 1) \bar{x} n(\bar{x}) + (\sigma_T^2 - 2\bar{\sigma}_S^2) \cdot \frac{N(\bar{x}) - N(x)}{2\bar{x}} \right]
\]

\[
+ \frac{\sigma_T^4 \gamma_T^2 n(\bar{x})}{\sigma_T^4} \left( 1 + \frac{2\gamma_T^2 n(\bar{x})}{\sigma_T^2} \cdot [N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})] \right) - C'_n(\sigma_n)
\]

Thus, for fixed $X$ and $\bar{X}$, when both tiger and fly are not very corrupt, society is worse off with the increase in the noise $\sigma_n$ before cost.
Case two: When both $\gamma_F$ and $\gamma_T$ are large, $X \to 0$, $\bar{X} \to 0$. From equation (66), Equation (57) can be expressed as:

\[
(67) \quad \frac{dE[W^s]}{d\sigma_n} \approx \frac{4\sigma_n \sigma_T^2 \bar{x}}{\sigma_I^4} \left[ (\beta^S - 1)\sigma_T^2 n(\bar{x}) + (\sigma_T^2 - 2\sigma_S^2) \frac{N(\bar{x}) - N(x)}{2\bar{x}} \right] - C'_n(\sigma_n)
\]

\[
\approx \frac{4\sigma_n \sigma_T^2 \bar{x} n(0)}{\sigma_I^4} \left[ (\beta^S - 1)\sigma_T^2 n(0) + (\sigma_T^2 - 2\sigma_S^2) n(0) \right] - C'_n(\sigma_n)
\]

\[
\approx \frac{4\sigma_n \sigma_T^2 \bar{x} n(0)}{\sigma_I^4} \left[ \beta^S \sigma_T^2 - \sigma_T^2 + 2\sigma_S^2 + \beta^T \gamma_T^2 + 2\gamma_T^2 - 2\beta^T \gamma_T^2 \right] - C'_n(\sigma_n)
\]

\[
= \frac{4\sigma_n \sigma_T^2 \bar{x} n(0)}{\sigma_I^4} \left[ \beta^S \sigma_T^2 - \beta^T \gamma_T^2 + 2\gamma_T^2 - 2\sigma_S^2 \right] - C'_n(\sigma_n)
\]

\[
= \frac{4\sigma_n \sigma_T^2 \bar{x} n(0)}{\sigma_I^4} \left[ \beta^T \sigma_T^2 \sigma_S^2 - \beta^T \gamma_T^2 + 2(\gamma_T^2 - \sigma_S^2) \right] - C'_n(\sigma_n)
\]

\[
= \frac{4\sigma_n \sigma_T^2 \bar{x} n(0)}{\sigma_I^4} \left[ \beta^T (\sigma_S^2 - \gamma_T^2) + 2(\gamma_T^2 - \sigma_S^2) \right] - C'_n(\sigma_n)
\]

Thus, for fixed $X$ and $\bar{X}$, when both tiger and fly are very corrupt, society is better off with the increase in the noise $\sigma_n$ before cost.
\[
\frac{d^2 E[W^S]}{d\gamma_F d\sigma_n} = \frac{d\sigma_F}{d\gamma_F} \left( \frac{\partial^2 E[W^S]}{\partial \sigma_F^2} \frac{d\sigma_F}{d\sigma_n} + \frac{2\partial^2 E[W^S]}{\partial \sigma_F \partial \dot{X}} \frac{d\dot{X}}{d\sigma_n} + \frac{\partial^2 E[W^S]}{\partial \sigma_F \partial \beta^I} \frac{d\beta^I}{d\sigma_n} \right) \\
+ \frac{2d\dot{X}}{d\gamma_F} \left( \frac{\partial^2 E[W^S]}{\partial \dot{X} \partial \sigma_F} \frac{d\sigma_F}{d\sigma_n} + \frac{\partial^2 E[W^S]}{\partial \dot{X}^2} \frac{d\dot{X}}{d\sigma_n} + \frac{\partial^2 E[W^S]}{\partial \dot{X} \partial \beta^I} \frac{d\beta^I}{d\sigma_n} \right) \\
+ \frac{dC(\gamma_F)}{d\gamma_F} \left( \frac{\partial^2 E[W^S]}{\partial C(\gamma_F) \partial \sigma_F} \frac{d\sigma_F}{d\sigma_n} + \frac{2\partial^2 E[W^S]}{\partial C(\gamma_F) \partial \dot{X}} \frac{d\dot{X}}{d\sigma_n} + \frac{\partial^2 E[W^S]}{\partial C(\gamma_F) \partial \beta^I} \frac{d\beta^I}{d\sigma_n} \right)
\]
Social Welfare, Tiger's utility and noise $\sigma_n$ with large $\gamma_T$ and small $\gamma_F$ ($\gamma_T = 3$, $\gamma_F = 0.1$)

Noise $\sigma_n$

Social Welfare and Tiger's utility
Social Welfare with cost
Social Welfare without cost

Tiger's utility

Social Welfare, Tiger's utility and noise $\sigma_n$ with small $\gamma_T$ and large $\gamma_F$ ($\gamma_T = 0.1$, $\gamma_F = 3$)

Noise $\sigma_n$

Social Welfare and Tiger's utility
Social Welfare with cost
Social Welfare without cost

Tiger's utility

52
References

