Online Appendix to "Tigers and Flies: Conflicts of Interest, Discretion and Expertise in a Hierarchy"

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Appendix C Both Tigers and Flies

In this section, we prove Theorem 3.1, which says that if the tiger and the fly both have great available corruption, fighting the fly's corruption alone reduces social welfare.

C.1 Social Welfare and γ_F

Proof of Theorem 3.1. First, we derive an expression for social welfare. Since S and F are jointly normal, both having mean 0, we can write

$$(25) \quad S = \beta^S F + \eta_S,$$

where η_S is independent of F, $\beta^S = cov(S,F)/var(F) = \beta^I \sigma_S^2/\sigma_F^2$, and $var(\eta_S) = \sigma_S^2 - (\beta^S)^2\sigma_F^2 = \sigma_S^2 - \beta^S\beta^I\sigma_S^2 = \sigma_S^2(1-\beta^S\beta^I)$.

Recall from (1) that $W^S = V^S - C_T(\gamma_T) - C_F(\gamma_F) - C_n(\sigma_n)$, where V^S is social welfare before cost, which is

$$(26) \quad V^S = -E[(X-S)^2] = -E[(X-(\beta^S F + \eta_S))^2]$$
$$= -E[(X-\beta^S F)^2 - 2\eta_S(X-\beta^S F) + \eta_S^2] = -E[(X-\beta^S F)^2] - var(\eta_S).$$

Now, $E[\eta_S(X-\beta^S F)] = 0$ because $E[\eta_S] = 0$, the fly's choice of X is a function of F, and η_S and F are independent. From (1), (12) and (26), we have the following expression for social welfare ignoring cost:

$$(27) \quad V^S = -E[(\pi(\sigma_F\varphi, \underline{X}, \bar{X}) - \beta^S F)^2] - var(\eta_S)$$
$$= \int_{\varphi=-\infty}^{\underline{X}/\sigma_F} -(\underline{X} - \beta^S\sigma_F\varphi)^2 n(\varphi)\,d\varphi + \int_{\varphi=\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} -(\sigma_F\varphi - \beta^S\sigma_F\varphi)^2 n(\varphi)\,d\varphi$$
$$+ \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} -(\bar{X} - \beta^S\sigma_F\varphi)^2 n(\varphi)\,d\varphi - var(\eta_S)$$
$$= \int_{\varphi=-\infty}^{\underline{X}/\sigma_F} (-\underline{X}^2 + 2\underline{X}\frac{\beta^I\sigma_S^2}{\sigma_F}\varphi - \frac{(\beta^I)^2\sigma_S^4}{\sigma_F^2}\varphi^2) n(\varphi)\,d\varphi$$

$$
\begin{aligned}
&+ \int_{\varphi=\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} (-\sigma_F^2\varphi^2 + 2\beta^I\sigma_S^2\varphi^2 - \frac{(\beta^I)^2\sigma_S^4}{\sigma_F^2}\varphi^2)n(\varphi)\,d\varphi \\
&+ \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} (-\bar{X}^2 + 2\bar{X}\frac{\beta^I\sigma_S^2}{\sigma_F}\varphi - \frac{(\beta^I)^2\sigma_S^4}{\sigma_F^2}\varphi^2)n(\varphi)\,d\varphi - \sigma_S^2 + \frac{(\beta^I)^2\sigma_S^4}{\sigma_F^2} \\
= &\int_{\varphi=-\infty}^{\underline{X}/\sigma_F} (-\underline{X}^2 + 2\underline{X}\frac{\beta^I\sigma_S^2}{\sigma_F}\varphi)n(\varphi)\,d\varphi + \int_{\varphi=\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} (-\sigma_F^2\varphi^2 + 2\beta^I\sigma_S^2\varphi^2)n(\varphi)\,d\varphi \\
&+ \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} (-\bar{X}^2 + 2\bar{X}\frac{\beta^I\sigma_S^2}{\sigma_F}\varphi)n(\varphi)\,d\varphi - \sigma_S^2
\end{aligned}
$$

Equation (27) expresses social welfare in terms of $\underline{X}$, $\bar{X}$, σ_F, β^I, and σ_S. In the model, σ_S is a primitive exogenous parameter, γ_F, γ_T and σ_n are exogenous social choices for fighting corruption and training the flies. For computing (14), we are varying γ_F but leaving σ_S, γ_T, and σ_n fixed, which also implies the derived coefficients $\beta^I = \sigma_T^2/(\sigma_T^2 + \sigma_n^2)$, $\sigma_T = \sqrt{\sigma_S^2 + \gamma_T^2}$ and $\sigma_I = \sqrt{\sigma_T^2 + \sigma_n^2}$ are fixed. To use the chain rule to compute $dW^S/d\gamma_F$, we use the following dependencies for variables that are not constant and depend directly or indirectly on γ_F:

(a) W^S depends on V^S and γ_F: see (1).
(b) V^S depends on $\underline{X}$, $\bar{X}$ and σ_F: see (27).
(c) $\underline{X}$, $\bar{X}$ depend on σ_F and β^T: see (25) and definitions $\underline{x} \equiv \underline{X}/\sigma_F$ and $\bar{x} \equiv \bar{X}/\sigma_F$.
(d) σ_F depends on γ_F: see (9).
(e) β^T depends on γ_F: see (11).

Then, the total derivative of social welfare with respect to γ_F is:

$$
\begin{aligned}
(28)\quad \frac{dW^S}{d\gamma_F} = &\frac{\partial V^S}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial V^S}{\partial \underline{X}}\left(\frac{\partial \underline{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial \underline{X}}{\partial \beta^T}\frac{d\beta^T}{d\gamma_F}\right) \\
&+ \frac{\partial V^S}{\partial \bar{X}}\left(\frac{\partial \bar{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial \bar{X}}{\partial \beta^T}\frac{d\beta^T}{d\gamma_F}\right) - \frac{dC_F(\gamma_F)}{d\gamma_F}
\end{aligned}
$$

where the terms correspond to the terms in (14) but have been expanded more.

I. Fixed $\underline{X}$ and $\bar{X}$

For now, we are fixing $\underline{X}$ and $\bar{X}$ throughout this section. Since $\sigma_F^2 = (\sigma_T^2)^2/\sigma_I^2 + \gamma_F^2$, then

$$(29)\quad \frac{d\sigma_F}{d\gamma_F} = \frac{d}{d\gamma_F}\left([(\sigma_T^2)^2/(\sigma_I^2+\gamma_F^2)]^{\frac{1}{2}}\right) = \frac{\gamma_F}{\sigma_F} > 0$$

From equations (27) and (28),

$$\begin{aligned}
(30)\quad \frac{\partial V^S}{\partial \sigma_F} &= \int_{\varphi=-\infty}^{\underline{X}/\sigma_F} -\frac{2\underline{X}\beta^I\sigma_S^2}{\sigma_F^2}\varphi n(\varphi)\,d\varphi + \int_{\varphi=\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} -2\sigma_F\varphi^2 n(\varphi)\,d\varphi \\
&\quad + \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} -\frac{2\bar{X}\beta^I\sigma_S^2}{\sigma_F^2}\varphi n(\varphi)\,d\varphi \\
&= 2\underline{X}\beta^S n(\varphi)|_{-\infty}^{\underline{X}/\sigma_F} - 2\sigma_F(-\varphi n(\varphi))|_{\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} - 2\sigma_F N(\varphi)|_{\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} + 2\bar{X}\beta^S n(\varphi)|_{\bar{X}/\sigma_F}^{+\infty} \\
&= 2\underline{X}\beta^S n(\underline{X}/\sigma_F) + 2\sigma_F\frac{\bar{X}}{\sigma_F}n(\bar{X}/\sigma_F) - 2\sigma_F\frac{\underline{X}}{\sigma_F}n(\underline{X}/\sigma_F) \\
&\quad - 2\sigma_F[N(\bar{X}/\sigma_F) - N(\underline{X}/\sigma_F)] - 2\bar{X}\beta^S n(\bar{X}/\sigma_F) \\
&= 2\sigma_F(\beta^S-1)[\underline{X}/\sigma_F n(\underline{X}/\sigma_F) - \bar{X}/\sigma_F n(\bar{X}/\sigma_F)] - 2\sigma_F[N(\bar{X}/\sigma_F) - N(\underline{X}/\sigma_F)] \\
&= 2\sigma_F(\beta^S-1)[\underline{x}n(\underline{x}) - \bar{x}n(\bar{x})] - 2\sigma_F[N(\bar{x}) - N(\underline{x})] \\
&= \underbrace{4\sigma_F(1-\beta^S)\bar{x}n(\bar{x}) - 2\sigma_F[2N(\bar{x})-1]}_{k_S(\bar{x})}
\end{aligned}$$

Let $k_S(\bar{x}) \equiv 4\sigma_F(1-\beta^S)\bar{x}n(\bar{x}) - 2\sigma_F[2N(\bar{x})-1]$, when $\bar{x} > 0$

$$\begin{aligned}
k_S'(\bar{x}) &= 2\sigma_F[2(1-\beta^S)(n(\bar{x}) - \bar{x}^2 n(\bar{x})) - 2n(\bar{x})] \\
&= 2\sigma_F[2n(\bar{x}) - 2\bar{x}^2 n(\bar{x}) - 2\beta^S n(\bar{x}) + 2\beta^S\bar{x}^2 n(\bar{x}) - 2n(\bar{x})] \\
&= 2\sigma_F[\underbrace{-2\beta^S n(\bar{x})}_{<0} + \underbrace{2(\beta^S-1)\bar{x}^2 n(\bar{x})}_{<0}] < 0
\end{aligned}$$

Thus, $k_S(\bar{x})$ is decreasing in $\bar{x}$ when $\bar{x} > 0$. When $\bar{x} \to 0$, $k_S(\bar{x}) \to 0$. Thus, for $0 < \bar{x} < +\infty$, $k_S'(\bar{x}) < 0$ implies that $k_S(\bar{x}) < 0$.

When γ_F is very large, $\bar{X} \to 0$, $\partial V^S/\partial\sigma_F < 0$. Therefore,

$$(31)\quad \frac{\partial V^S}{\partial \sigma_F}|_{\underline{X},\bar{X}}\frac{d\sigma_F}{d\gamma_F} = 2\gamma_F(\beta^S-1)[\underline{x}n(\underline{x}) - \bar{x}n(\bar{x})] - 2\gamma_F[N(\bar{x}) - N(\underline{x})] < 0$$

Thus, when both $\underline{X}$ and $\bar{X}$ are fixed, reducing the fly's available corruption (γ_F) makes the society better off when γ_F is very large.

II. Tiger Chooses $\underline{X}$ and $\bar{X}$ Optimally in Response to γ_F

Now, we consider what happens to social welfare with changes in γ_F when the tiger chooses $\underline{X}$ and $\bar{X}$ optimally in response to γ_F.

$$(32)\quad \frac{\partial V^S}{\partial \bar{X}}\Big|_{\sigma_F,\underline{X}} = \int_{\bar{X}/\sigma_F}^{+\infty} (-2\bar{X} + 2\frac{\beta^I\sigma_S^2}{\sigma_F}\varphi) n(\varphi)\, d\varphi$$
$$= -2\bar{X}N(-\bar{X}/\sigma_F) + 2\frac{\beta^I\sigma_S^2}{\sigma_F} n(\bar{X}/\sigma_F)$$
$$= -2\sigma_F n(\bar{X}/\sigma_F)\left[\frac{(\bar{X}/\sigma_F)N(-\bar{X}/\sigma_F)}{n(\bar{X}/\sigma_F)} - \frac{\beta^I\sigma_S^2}{\sigma_F^2}\right]$$
$$= -\frac{2\beta^I\gamma_T^2 n(\bar{X}/\sigma_F)}{\sigma_F} = -\frac{2\beta^I\gamma_T^2 n(\bar{x})}{\sigma_F}$$

From equation (25) and the definition $\bar{x} \equiv \bar{X}/\sigma_F$,

$$\frac{\partial \beta^T}{\partial \bar{X}}\Big|_{\sigma_F} = \frac{\partial}{\partial \bar{X}}\left\{\frac{(\bar{X}/\sigma_F)N(-\bar{X}/\sigma_F)}{n(\bar{X}/\sigma_F)}\right\}$$
$$= \frac{N(-\bar{X}/\sigma_F)}{\sigma_F n(\bar{X}/\sigma_F)} + \frac{-\bar{X}n(\bar{X}/\sigma_F)}{\sigma_F^2 n(\bar{X}/\sigma_F)} + \frac{\bar{X}^2 N(-\bar{X}/\sigma_F)n(\bar{X}/\sigma_F)}{\sigma_F^3 n^2(\bar{X}/\sigma_F)}$$
$$= \frac{1}{\sigma_F n(\underline{X}/\sigma_F)}\left(N(-\bar{X}/\sigma_F) - \frac{\bar{X}}{\sigma_F} n(\bar{X}/\sigma_F) + \frac{\bar{X}^2}{\sigma_F^2} N(-\bar{X}/\sigma_F)\right)$$
$$= \frac{1}{\sigma_F n(\underline{x})}[N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})]$$

Thus,

$$(33)\quad \frac{\partial \bar{X}}{\partial \beta^T}\Big|_{\sigma_F} = \frac{\sigma_F n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-x)}$$

When β^T is fixed, $\bar{X}/\sigma_F$ is also fixed. Thus, $\partial(\bar{X}/\sigma_F) = 0$, $\partial log(\bar{X}/\sigma_F) =$

$\partial log(\bar{X}) - \partial log(\sigma_F) = 0$, $\partial \bar{X}/\bar{X} = \partial\sigma_F/\sigma_F$, and

$$(34)\quad \left.\frac{\partial \bar{X}}{\partial \sigma_F}\right|_{\beta^T} = \frac{\bar{X}}{\sigma_F} = \bar{x}$$

$$(35)\quad \frac{d\beta^T}{d\gamma_F} = \frac{d}{d\gamma_F}\left(\frac{\beta^I\sigma_T^2}{\beta^I\sigma_T^2+\gamma_F^2}\right) = -\frac{2\gamma_F\beta^I\sigma_T^2}{(\beta^I\sigma_T^2+\gamma_F^2)^2} = -\frac{2\gamma_F\beta^T}{\sigma_F^2}$$

$$\begin{aligned}(36)\quad & \frac{\partial V^S}{\partial \bar{X}}\left(\frac{\partial \bar{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial \bar{X}}{\partial \beta^T}\frac{d\beta^T}{d\gamma_F}\right) \\ & = -\frac{2\beta^I\gamma_T^2 n(\bar{x})}{\sigma_F}\left[\bar{x}\frac{\gamma_F}{\sigma_F} + \frac{\sigma_F n(\bar{x})}{N(-\bar{x})-\bar{x}n(\bar{x})+\bar{x}^2N(-x)}\left(-\frac{2\gamma_F\beta^T}{\sigma_F^2}\right)\right] \\ & = -\frac{2\beta^I\gamma_T^2\gamma_F n(\bar{x})}{\sigma_F^2}\left[\bar{x} - \frac{2\beta^T n(\bar{x})}{N(-\bar{x})-\bar{x}n(\bar{x})+\bar{x}^2N(-x)}\right] \\ & = \frac{2\beta^I\gamma_F\gamma_T^2 n(\bar{x})}{\sigma_F^2}\left[\frac{2\beta^T n(\bar{x})}{N(-\bar{x})-\bar{x}n(\bar{x})+\bar{x}^2N(-\bar{x})} - \bar{x}\right]\end{aligned}$$

Similarly,

$$\begin{aligned}(37)\quad & \frac{\partial V^S}{\partial \underline{X}}\left(\frac{\partial \underline{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial \underline{X}}{\partial \beta^T}\frac{d\beta^T}{d\gamma_F}\right) = \frac{\partial V^S}{\partial \bar{X}}\left(\frac{\partial \bar{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial \bar{X}}{\partial \beta^T}\frac{d\beta^T}{d\gamma_F}\right) \\ & = \frac{2\beta^I\gamma_F\gamma_T^2 n(\bar{x})}{\sigma_F^2}\left[\frac{2\beta^T n(\bar{x})}{N(-\bar{x})-\bar{x}n(\bar{x})+\bar{x}^2N(-\bar{x})} - \bar{x}\right]\end{aligned}$$

When both γ_T and γ_F are very large, $\underline{X} \to 0$, $\bar{X} \to 0$, $\bar{x} \to 0$; $\beta^S \to 0$, $\beta^T \to 0$, $C_F(\gamma_F) \to 0$, $C'_F(\gamma_F) \to 0$. Since $f'(x) \equiv \lim_{\Delta\to 0}\frac{f(x+\Delta)-f(x)}{\Delta}$, and $f'(x) \equiv \lim_{\Delta\to 0}\frac{f(x+\Delta)-f(x-\Delta)}{2\Delta}$, we have

$$(38)\quad \lim_{\bar{x}\to 0}\frac{N(\bar{x})-N(\underline{x})}{2\bar{x}} = n(0)$$

$$(39)\quad \lim_{\gamma_F\uparrow+\infty}\frac{\gamma_F^2}{\sigma_F^2} = \lim_{\gamma_F\uparrow+\infty} 1-\beta^T = \lim_{\gamma_F\uparrow+\infty} 1 - \frac{(\bar{X}/\sigma_F)N(-\bar{X}/\sigma_F)}{n(\bar{X}/\sigma_F)}$$

$$= \lim_{\gamma_F \uparrow +\infty} 1 - \frac{(\bar{X}/\sigma_F)(1/2 - n(0)\bar{X}/\sigma_F)}{n(0)} = \lim_{\gamma_F \uparrow +\infty} 1 - \frac{1}{2n(0)}\left(\frac{\bar{X}}{\sigma_F}\right) + \left(\frac{\bar{X}}{\sigma_F}\right)^2$$

$$= \lim_{\gamma_F \uparrow +\infty} 1 - \frac{1}{2n(0)}\left(\frac{\bar{X}}{\sigma_F}\right) = 1 - \frac{\bar{x}}{2n(0)}$$

$$(40) \quad 0 < \frac{\beta^I \gamma_T^2}{\sigma_F^2} = \frac{\beta^I(\sigma_T^2 - \sigma_S^2)}{\beta^I \sigma_T^2 + \gamma_F^2} < 1$$

From Equations (31), (38), (39) and (40),

$$\begin{aligned}
(41) \quad \frac{dW^S}{d\gamma_F} &= 2\gamma_F(\beta^S - 1)[\underline{x}n(\underline{x}) - \bar{x}n(\bar{x})] - 2\gamma_F[N(\bar{x}) - N(\underline{x})] - C_F'(\gamma_F) \\
&\quad + 2\left(\frac{2\beta^I \gamma_F \gamma_T^2 n(\bar{x})}{\sigma_F^2}\right)\left[\frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-x)} - \bar{x}\right] \\
&= 4\gamma_F \bar{x}\left[(1 - \beta^S)n(\bar{x}) - \frac{N(\bar{x}) - N(\underline{x})}{2\bar{x}}\right. \\
&\quad \left. + \frac{\beta^I \gamma_T^2 n(\bar{x})}{\sigma_F^2}\left(\frac{2}{1 - \bar{x}n(x)/N(-\bar{x}) + \bar{x}^2} - 1\right)\right] - C_F'(\gamma_F) \\
&\approx 4\gamma_F \bar{x}\left[(1 - \beta^S)n(\bar{x}) - \frac{N(\bar{x}) - N(\underline{x})}{2\bar{x}} + \frac{\beta^I \gamma_T^2 n(\bar{x})}{\sigma_F^2}\left(\underbrace{\frac{2}{p(-\bar{x})} - 1}_{\to 1}\right)\right] - C_F'(\gamma_F) \\
&= 4\gamma_F \bar{x}\left[(1 - \beta^S)n(0) - n(0) + \frac{\beta^I \gamma_T^2 n(0)}{\sigma_F^2}\right] - C_F'(\gamma_F) \\
&= 4\gamma_F \bar{x}\left[-\beta^S n(0) + \frac{\beta^I \gamma_T^2 n(0)}{\sigma_F^2}\right] - C_F'(\gamma_F) \\
&= 4\gamma_F \bar{x}\left[-\frac{\beta^I \sigma_S^2}{\sigma_F^2} n(0) + \frac{\beta^I \gamma_T^2 n(0)}{\sigma_F^2}\right] - C_F'(\gamma_F) \\
&= 4\gamma_F \bar{x}\left[\underbrace{\frac{\beta^I n(0)}{\sigma_F^2}(\gamma_T^2 - \sigma_S^2)}_{>0 \text{ when } \gamma_T \text{ is large enough}}\right] - C_F'(\gamma_F)
\end{aligned}$$

When both γ_F and γ_F are large enough, social welfare is increasing in γ_F. Thus, when the fly and the tiger both have large available corruptions, fighting the fly's corruption γ_F without fighting the tiger's corruption γ_T makes the society worse off. This is because the damage done by the tiger increasing the fly's discretion (increasing $\bar{x}$ or reducing $\underline{x}$) more than offsets the direct benefits of reducing the fly's available corruption given $\bar{x}$ and $\underline{x}$. This proves Theorem 3.1. ■

C.2 Tiger and γ_F

In this section, we prove Theorem 3.2, which says that the tiger always prefers to fight the fly's corruption more than is socially optimal. It is because the tiger receives more than proportionally the benefits but bears less than proportionally the costs.

Proof of Theorem 3.2. First, we derive an expression for the tiger's utility. Based on equation (22), the tiger's utility is

$$
\begin{aligned}
(42)\quad U^T &= \gamma_T^2 - E[(\pi(\sigma_F\varphi, \underline{X}, \bar{X}) - \beta^T F)^2] - var(\eta_T) \\
&= \gamma_T^2 + \int_{\varphi=-\infty}^{\underline{X}/\sigma_F} -(\underline{X} - \beta^T\sigma_F\varphi)^2 n(\varphi)\, d\varphi + \int_{\varphi=\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} -(\sigma_F\varphi + \beta^T\sigma_F\varphi)^2 n(\varphi)\, d\varphi \\
&\quad + \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} -(\bar{X} + \beta^T\sigma_F\varphi)^2 n(\varphi)\, d\varphi - var(\eta_T) \\
&= \gamma_T^2 + \int_{\varphi=-\infty}^{\underline{X}/\sigma_F} (-\underline{X}^2 + 2\underline{X}\frac{\beta^I\sigma_T^2}{\sigma_F}\varphi - \frac{(\beta^I)^2\sigma_T^4}{\sigma_F^2}\varphi^2) n(\varphi)\, d\varphi \\
&\quad + \int_{\varphi=\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} (-\sigma_F^2\varphi^2 + 2\beta^I\sigma_T^2\varphi^2 - \frac{(\beta^I)^2\sigma_T^4}{\sigma_F^2}\varphi^2) n(\varphi)\, d\varphi \\
&\quad + \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} (-\bar{X}^2 + 2\bar{X}\frac{\beta^I\sigma_T^2}{\sigma_F}\varphi - \frac{(\beta^I)^2\sigma_T^4}{\sigma_F^2}\varphi^2) n(\varphi)\, d\varphi - \sigma_T^2 + \frac{(\beta^I)^2\sigma_T^4}{\sigma_F^2} \\
&= \gamma_T^2 + \int_{\varphi=-\infty}^{\underline{X}/\sigma_F} (-\underline{X}^2 + 2\underline{X}\frac{\beta^I\sigma_T^2}{\sigma_F}\varphi) n(\varphi)\, d\varphi + \int_{\varphi=\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} (-\sigma_F^2\varphi^2 + 2\beta^I\sigma_T^2\varphi^2) n(\varphi)\, d\varphi \\
&\quad + \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} (-\bar{X}^2 + 2\bar{X}\frac{\beta^I\sigma_T^2}{\sigma_F}\varphi) n(\varphi)\, d\varphi - \sigma_T^2
\end{aligned}
$$

Equation (42) expresses the tiger's utility in terms of $\underline{X}$, $\bar{X}$, σ_F, β^I, σ_T and

γ_T. In the model, γ_F, γ_T and σ_n are exogenous social choices for fighting corruption and training the flies. For computing (15), we are varying γ_F but leaving σ_S, γ_T, and σ_n fixed, which also implies the derived coefficients $\beta^I = \sigma_T^2/(\sigma_T^2 + \sigma_n^2)$, $\sigma_T = \sqrt{\sigma_S^2 + \gamma_T^2}$ and $\sigma_I = \sqrt{\sigma_T^2 + \sigma_n^2}$ are fixed. To use the chain rule to compute $dU^T/d\gamma_F$, we use the following dependencies for variables that are not constant and depend directly or indirectly on γ_F:

(a) U^T depends on $\underline{X}$, $\bar{X}$ and σ_F: see (42).
(b) $\underline{X}$, $\bar{X}$ depend on σ_F and β^T: see (25) and definitions $\underline{x} \equiv \underline{X}/\sigma_F$ and $\bar{x} \equiv \bar{X}/\sigma_F$.
(c) σ_F depends on γ_F: see (9).
(d) β^T depends on γ_F: see (11).

Then, the total derivative of the tiger's utility with respect to γ_F is:

$$\text{(43)}\quad \frac{dU^T}{d\gamma_F} = \frac{\partial U^T}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial U^T}{\partial \underline{X}}\left(\frac{\partial \underline{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial \underline{X}}{\partial \beta^T}\frac{d\beta^T}{d\gamma_F}\right) + \frac{\partial U^T}{\partial \bar{X}}\left(\frac{\partial \bar{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial \bar{X}}{\partial \beta^T}\frac{d\beta^T}{d\gamma_F}\right)$$

where the terms correspond to the terms in (15) but have been expanded more.

$$\begin{aligned}
\text{(44)}\quad \left.\frac{\partial U^T}{\partial \sigma_F}\right|_{\underline{X},\bar{X}} &= \int_{\varphi=-\infty}^{\underline{X}/\sigma_F} -\frac{2\underline{X}\beta^I\sigma_T^2}{\sigma_F^2}\varphi n(\varphi)\,d\varphi + \int_{\varphi=\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} -2\sigma_F\varphi^2 n(\varphi)\,d\varphi \\
&\quad + \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} -\frac{2\bar{X}\beta^I\sigma_T^2}{\sigma_F^2}\varphi n(\varphi)\,d\varphi \\
&= 2\underline{X}\beta^T n(\varphi)|_{-\infty}^{\underline{X}/\sigma_F} - 2\sigma_F(-\varphi n(\varphi))|_{\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} - 2\sigma_F N(\varphi)|_{\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} + 2\bar{X}\beta^T n(\varphi)|_{\bar{X}/\sigma_F}^{+\infty} \\
&= 2\underline{X}\beta^T n(\underline{X}/\sigma_F) + 2\sigma_F\frac{\bar{X}}{\sigma_F}n(\bar{X}/\sigma_F) - 2\sigma_F\frac{\underline{X}}{\sigma_F}n(\underline{X}/\sigma_F) \\
&\quad - 2\sigma_F[N(\bar{X}/\sigma_F) - N(\underline{X}/\sigma_F)] - 2\bar{X}\beta^T n(\bar{X}/\sigma_F) \\
&= 2\sigma_F(\beta^T - 1)[\underline{X}/\sigma_F n(\underline{X}/\sigma_F) - \bar{X}/\sigma_F n(\bar{X}/\sigma_F)] - 2\sigma_F[N(\bar{X}/\sigma_F) - N(\underline{X}/\sigma_F)] \\
&= 2\sigma_F(\beta^T - 1)[\underline{x}n(\underline{x}) - \bar{x}n(\bar{x})] - 2\sigma_F[N(\bar{x}) - N(\underline{x})] \\
&= \underbrace{4\sigma_F(1-\beta^T)\bar{x}n(\bar{x}) - 2\sigma_F[2N(\bar{x}) - 1]}_{k_T(\bar{x})}
\end{aligned}$$

Let $k_T(\bar{x}) \equiv 4\sigma_F(1-\beta^T)\bar{x}n(\bar{x}) - 2\sigma_F[2N(\bar{x})-1]$, when $\bar{x} > 0$

$$
\begin{aligned}
(45)\quad k_T'(\bar{x}) &= 2\sigma_F[2(1-\beta^T)(n(\bar{x}) - \bar{x}^2 n(\bar{x})) - 2n(\bar{x})] \\
&= 2\sigma_F[2n(\bar{x}) - 2\bar{x}^2 n(\bar{x}) - 2\beta^T n(\bar{x}) + 2\beta^T \bar{x}^2 n(\bar{x}) - 2n(\bar{x})] \\
&= 2\sigma_F[\underbrace{-2\beta^T n(\bar{x})}_{<0} + \underbrace{2(\beta^T - 1)\bar{x}^2 n(\bar{x})}_{<0}] < 0
\end{aligned}
$$

Thus, $k_T(\bar{x})$ is decreasing with $\bar{x}$ when $\bar{x} > 0$. When $\bar{x} \to 0$, $k_T(\bar{x}) \to 0$. Thus, for $0 < \bar{x} < +\infty$, $k_T'(\bar{x}) < 0$ implies that $k_T(\bar{x}) < 0$. From (29), (44) and (45), we have

$$
(46)\quad \left.\frac{\partial U^T}{\partial \sigma_F}\right|_{\underline{X},\bar{X}} \frac{d\sigma_F}{d\gamma_F} = 2\gamma_F(\beta^T - 1)[\underline{x}n(\underline{x}) - \bar{x}n(\bar{x})] - 2\gamma_F[N(\bar{x}) - N(\underline{x})] < 0
$$

For fixed $\underline{X}$ and $\bar{X}$, the tiger's utility is decreasing in γ_F. Also, this is true if the tiger chooses $\underline{X}$ and $\bar{X}$, since the tiger will choose $\underline{X}$ and $\bar{X}$ to maximize utility and the pointwise maximum of decreasing functions is decreasing.

■

Appendix D Information and Discretion

D.1 $\bar{X}$ and γ_F

In this section, we prove Theorem 4.1, which says that fighting corruption and imposing stringent rules are substitutes from the tiger's perspective.

Proof of Theorem 4.1. From Equations (13), (29), (33), (34), (35),

$$
\begin{aligned}
(47)\quad \frac{d\bar{X}}{d\gamma_F} &= \frac{\partial \bar{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial \bar{X}}{\partial \beta^T}\frac{d\beta^T}{d\gamma_F} \\
&= \bar{x}\frac{\gamma_F}{\sigma_F} + \frac{\sigma_F n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-x)}\left(-\frac{2\gamma_F\beta^T}{\sigma_F^2}\right)
\end{aligned}
$$

$$= \frac{\gamma_F}{\sigma_F}\left(\bar{x} - \frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})}\right)$$
$$= \frac{\gamma_F \bar{x}}{\sigma_F}\left(1 - \frac{2N(-\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})}\right)$$
$$= \frac{\gamma_F \bar{x}}{\sigma_F}\left[1 - \frac{2}{1 - \bar{x}n(-\bar{x})/N(-\bar{x}) + (-\bar{x})^2}\right]$$
$$= \frac{\gamma_F \bar{x}}{\sigma_F}\left(1 - \frac{2}{p(-\bar{x})}\right) < 0$$

The final inequality follows from Lemma A1(d), which defines p(x) and implies $p(x) \in (0, 1)$, so $2/p(x) \in (2, +\infty)$. Similarly,

$$\frac{d\underline{X}}{d\gamma_F} = -\frac{d\bar{X}}{d\gamma_F} > 0 \tag{48}$$

Thus, discretion is decreasing in the fly's available corruption γ_F. This proves Theorem 4.1. ■

D.2 Fly and γ_F

In this section, we prove Theorem 4.2, which says that fighting the fly's corruption makes the fly worse off when γ_F is small enough and better off when γ_F is large enough.

Proof of Theorem 4.2. First, we derive an expression for the fly's utility. Based on (7), the fly's utility is

$$\begin{aligned} U^F &= \gamma_F^2 + E[-(\pi(\sigma_F\varphi, \underline{X}, \bar{X}) - \sigma_F\varphi)^2] - var(\eta_I) \\ &= \gamma_F^2 + \int_{\varphi=-\infty}^{\underline{X}/\sigma_F} -(\underline{X} - \sigma_F\varphi)^2 n(\varphi)\, d\varphi + \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} -(\bar{X} - \sigma_F\varphi)^2 n(\varphi)\, d\varphi - \sigma_T^2(1-\beta^I) \\ &= \sigma_F^2 - \beta^I\sigma_T^2 + \int_{\varphi=-\infty}^{\underline{X}/\sigma_F} (-\underline{X}^2 + 2\underline{X}\sigma_F\varphi - \sigma_F^2\varphi^2) n(\varphi)\, d\varphi \\ &\quad + \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} (-\bar{X}^2 + 2\bar{X}\sigma_F\varphi - \sigma_F^2\varphi^2) n(\varphi)\, d\varphi - \sigma_T^2(1-\beta^I) \end{aligned} \tag{49}$$

$$
\begin{aligned}
&= \sigma_F^2 - \sigma_T^2 + \int_{\varphi=-\infty}^{\underline{X}/\sigma_F} (-\underline{X}^2 + 2\underline{X}\sigma_F\varphi - \sigma_F^2\varphi^2) n(\varphi)\, d\varphi \\
&\quad + \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} (-\bar{X}^2 + 2\bar{X}\sigma_F\varphi - \sigma_F^2\varphi^2) n(\varphi)\, d\varphi
\end{aligned}
$$

Equation (49) expresses the fly's utility in terms of $\underline{X}$, $\bar{X}$, σ_F, and σ_T. In the model, γ_F, γ_T and σ_n are exogenous social choices for fighting corruption and training the flies. For computing (17), we are varying γ_F but leaving σ_S, γ_T, and σ_n fixed, which also implies the derived coefficients $\beta^I = \sigma_T^2/(\sigma_T^2 + \sigma_n^2)$, $\sigma_T = \sqrt{\sigma_S^2 + \gamma_T^2}$ and $\sigma_I = \sqrt{\sigma_T^2 + \sigma_n^2}$ are fixed. To use the chain rule to compute $dU^F/d\gamma_F$, we use the following dependencies for variables that are not constant and depend directly or indirectly on γ_F:

(a) U^F depends on $\underline{X}$, $\bar{X}$ and σ_F: see (49).
(b) $\underline{X}$, $\bar{X}$ depend on σ_F and β^T: see (25) and definitions $\underline{x} \equiv \underline{X}/\sigma_F$ and $\bar{x} \equiv \bar{X}/\sigma_F$.
(c) σ_F depends on γ_F: see (9).
(d) β^T depends on γ_F: see (11).

Thus, the total derivative of the fly's utility with respect to γ_F is:

$$
(50) \quad \frac{dU^F}{d\gamma_F} = \frac{\partial U^F}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial U^F}{\partial \underline{X}}\left(\frac{\partial \underline{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial \underline{X}}{\partial \beta^T}\frac{d\beta^T}{d\gamma_F}\right) + \frac{\partial U^F}{\partial \bar{X}}\left(\frac{\partial \bar{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial \underline{X}}{\partial \beta^T}\frac{d\beta^T}{d\gamma_F}\right)
$$

where the terms correspond to the terms in (17) but have been expanded more.

I. Fixed $\underline{X}$ and $\bar{X}$

For now, we are fixing $\underline{X}$ and $\bar{X}$ throughout this section. From Equations (49),

$$
\begin{aligned}
(51) \quad \frac{\partial U^F}{\partial \sigma_F}|_{\underline{X},\bar{X}} &= 2\sigma_F + \left(-\frac{\underline{X}}{\sigma_F^2}\right)(-\underline{X}^2 + 2\underline{X}\sigma_F\frac{\underline{X}}{\sigma_F} - \sigma_F^2\frac{\underline{X}^2}{\sigma_F^2}) n(\underline{X}/\sigma_F) \\
&\quad + \int_{\varphi=-\infty}^{\underline{X}/\sigma_F} \left(2\underline{X}\varphi - 2\sigma_F\varphi^2\right) n(\varphi)\, d\varphi
\end{aligned}
$$

$$
\begin{aligned}
&+ \left(\frac{\bar{X}}{\sigma_F^2}\right)(-\bar{X}^2 + 2\bar{X}\sigma_F\frac{\bar{X}}{\sigma_F} - \sigma_F^2\frac{\bar{X}^2}{\sigma_F^2})n(\bar{X}/\sigma_F) \\
&+ \int_{\varphi=\bar{X}/\sigma_F}^{+\infty}\Big(2\bar{X}\varphi - 2\sigma_F\varphi^2\Big)n(\varphi)\,d\varphi \\
= {}& 2\sigma_F + \int_{\varphi=-\infty}^{\frac{\underline{X}}{\sigma_F}}\Big(2\underline{X}\varphi - 2\sigma_F\varphi^2\Big)n(\varphi)\,d\varphi + \int_{\varphi=\bar{X}/\sigma_F}^{+\infty}\Big(2\bar{X}\varphi - 2\sigma_F\varphi^2\Big)n(\varphi)\,d\varphi \\
= {}& 2\sigma_F + 2\underline{X}[-n(\varphi)]|_{-\infty}^{\underline{X}/\sigma_F} - 2\sigma_F\int_{\varphi=-\infty}^{\underline{X}/\sigma_F}(n''(\varphi) + n(\varphi))\,d\varphi \\
&+ 2\bar{X}[-n(\varphi)]|_{\varphi=\bar{X}/\sigma_F}^{+\infty} - 2\sigma_F\int_{\varphi=\bar{X}/\sigma_F}^{+\infty}(n''(\varphi) + n(\varphi))\,d\varphi \\
= {}& 2\sigma_F - 2\underline{X}n(\underline{X}/\sigma_F) + 2\sigma_F(\underline{X}/\sigma_F)n(\underline{X}/\sigma_F) - 2\sigma_F N(\underline{X}/\sigma_F) \\
&+ 2\bar{X}n(\bar{X}/\sigma_F) - 2\sigma_F(\bar{X}/\sigma_F)n(\bar{X}/\sigma_F) - 2\sigma_F(1 - N(\bar{X}/\sigma_F)) \\
= {}& 2\sigma_F N(\bar{X}/\sigma_F) - 2\sigma_F N(\underline{X}/\sigma_F) = 2\sigma_F[N(\bar{x}) - N(\underline{x})] > 0
\end{aligned}
$$

From Equations (29), (50), and (51),

$$
(52)\quad \frac{\partial U^F}{\partial \sigma_F}|_{\underline{X},\bar{X}}\frac{d\sigma_F}{d\gamma_F} = 2\sigma_F[N(\bar{x}) - N(\underline{x})]\frac{\gamma_F}{\sigma_F} = 2\gamma_F[N(\bar{x}) - N(\underline{x})] > 0
$$

Therefore, when $\underline{X}$ and $\bar{X}$ are fixed, the fly's utility is increasing in γ_F.

II. Tiger Chooses $\underline{X}$ and $\bar{X}$ Optimally in Response to γ_F

Now we consider what happens when the tiger chooses $\underline{X}$ and $\bar{X}$ optimally in response to γ_F,

$$
\begin{aligned}
(53)\quad \left.\frac{\partial U^F}{\partial \bar{X}}\right|_{\sigma_F,\underline{X}} &= -\frac{1}{\sigma_F}(-\bar{X}^2 + 2\bar{X}^2 - \bar{X}^2)n(\bar{X}/\sigma_F) + \int_{\varphi=\bar{X}/\sigma_F}^{+\infty}(-2\bar{X} + 2\sigma_F\varphi)n(\varphi)\,d\varphi \\
&= -2\bar{X}N(\varphi)|_{\bar{X}/\sigma_F}^{+\infty} - 2\sigma_F n(\varphi)|_{\bar{X}/\sigma_F}^{+\infty} \\
&= -2\bar{X}N(-\bar{X}/\sigma_F) + 2\sigma_F n(\bar{X}/\sigma_F) \\
&= -2\sigma_F n(\bar{X}/\sigma_F)\left(\frac{\bar{X}/\sigma_F N(-\bar{X}/\sigma_F)}{n(\bar{X}/\sigma_F)} - 1\right) \\
&= -2\sigma_F n(\bar{X}/\sigma_F)(\beta^T - 1) = \frac{2\gamma_F^2 n(\bar{x})}{\sigma_F}
\end{aligned}
$$

$$(54) \quad \frac{\partial U^F}{\partial \bar{X}}\left(\frac{\partial \bar{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial \underline{X}}{\partial \beta^T}\frac{d\beta^T}{d\gamma_F}\right)$$

$$= \frac{2\gamma_F^2 n(\bar{x})}{\sigma_F}\left[\bar{x}\frac{\gamma_F}{\sigma_F} + \frac{\sigma_F n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-x)}\left(-\frac{2\gamma_F \beta^T}{\sigma_F^2}\right)\right]$$

$$= \frac{2\gamma_F^3 n(\bar{x})}{\sigma_F^2}\left[\bar{x} - \frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})}\right]$$

$$= -\frac{2\gamma_F^3 n(\bar{x})}{\sigma_F^2}\left[\frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} - \bar{x}\right]$$

Similarly,

$$(55) \quad \frac{\partial U^F}{\partial \underline{X}}\left(\frac{\partial \underline{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial \underline{X}}{\partial \beta^T}\frac{d\beta^T}{d\gamma_F}\right) = \frac{\partial U^F}{\partial \bar{X}}\left(\frac{\partial \bar{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\gamma_F} + \frac{\partial \underline{X}}{\partial \beta^T}\frac{d\beta^T}{d\gamma_F}\right)$$

$$= -\frac{2\gamma_F^3 n(\bar{x})}{\sigma_F^2}\left[\frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} - \bar{x}\right]$$

When γ_F is very large, $\underline{X} \to 0$, $\bar{X} \to 0$. From Equations (39), (52), (54), and (55), Equation (50) can be expressed as

$$(56) \quad \frac{dU^F}{d\gamma_F} = 2\gamma_F[N(\bar{x}) - N(\underline{x})] - \frac{4\gamma_F^3 n(\bar{x})}{\sigma_F^2}\left[\frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} - \bar{x}\right]$$

$$= 4\bar{x}^2\gamma_F\left[\frac{N(\bar{x}) - N(-\bar{x})}{2\bar{x}^2} - \frac{\gamma_F^2}{\sigma_F^2}\frac{n(\bar{x})}{\bar{x}}\left(\frac{2}{1 - \bar{x}n(\bar{x})/N(-\bar{x}) + \bar{x}^2} - 1\right)\right]$$

$$= 4\bar{x}^2\gamma_F\left[\frac{N(\bar{x}) - N(-\bar{x})}{2\bar{x}^2} - \frac{\gamma_F^2}{\sigma_F^2}\frac{n(\bar{x})}{\bar{x}}\left(\frac{2}{p(-\bar{x})} - 1\right)\right]$$

$$\approx 4\bar{x}^2\gamma_F\left[\frac{n(0)}{\bar{x}} - \left(1 - \frac{1}{2n(0)}\bar{x}\right)\frac{n(0)}{\bar{x}}\left(\left(2 + \frac{2n(0)}{N(0)}\bar{x}\right) - 1\right)\right]$$

$$= 4\bar{x}^2\gamma_F[\underbrace{-4n(0)^2 + \frac{1}{2}}_{<0} + \underbrace{2n(0)\bar{x}}_{\to 0}] < 0$$

Therefore, the fly's utility is decreasing in γ_F when γ_F is large.

When γ_F is small, $\underline{X} \to -\infty$, $\bar{X} \to +\infty$ and $\beta^T \to 1$.

$$(57) \quad \lim_{\gamma_F \downarrow 0}\frac{\gamma_F^2}{\sigma_F^2} = \lim_{\gamma_F \downarrow 0}\frac{\gamma_F^2}{\beta^I\sigma_T^2 + \gamma_F^2} = \lim_{\gamma_F \downarrow 0}\frac{1}{\beta^I\sigma_T^2/\gamma_F^2 + 1} = 0$$

Equation (50) can be written as

$$\begin{aligned}
(58)\quad \frac{dU^F}{d\gamma_F} &= 2\gamma_F\left[N(\bar{x}) - N(-\bar{x}) - \frac{\gamma_F^2}{\sigma_F^2}n(\bar{x})\left(\frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} - \bar{x}\right)\right] \\
&= 2\gamma_F\left[N(\bar{x}) - N(-\bar{x}) - \frac{\gamma_F^2}{\sigma_F^2}n(\bar{x})\left(\frac{2\bar{x}N(-\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} - \bar{x}\right)\right] \\
&= 2\gamma_F\left[N(\bar{x}) - N(-\bar{x}) - \frac{\gamma_F^2}{\sigma_F^2}n(\bar{x})\bar{x}^2\frac{1}{\bar{x}^2}\left(\frac{N(-\bar{x}) + \bar{x}n(\bar{x}) - \bar{x}^2 N(-\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})}\right)\right]
\end{aligned}$$

As a preliminary, let's consider the expression:

$$\begin{aligned}
(59)\quad & \frac{1}{\bar{x}^2}\frac{N(-\bar{x}) + \bar{x}n(\bar{x}) - \bar{x}^2 N(-\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \\
&= \frac{1}{\bar{x}^2}\left\{\frac{\left[\frac{n(\bar{x})}{\bar{x}}\left(1 - \frac{1}{\bar{x}^2} + R_1\right)\right] + \bar{x}n(\bar{x}) - \bar{x}^2\left[\frac{n(\bar{x})}{\bar{x}}\left(1 - \frac{1}{\bar{x}^2} + \frac{3}{\bar{x}^4} + R_2\right)\right]}{\left[\frac{n(\bar{x})}{\bar{x}}\left(1 - \frac{1}{\bar{x}^2} + R_1\right)\right] - \bar{x}n(\bar{x}) + \bar{x}^2\left[\frac{n(\bar{x})}{\bar{x}}\left(1 - \frac{1}{\bar{x}^2} + \frac{3}{\bar{x}^4} + R_2\right)\right]}\right\} \\
&= \frac{1}{\bar{x}^2}\left\{\frac{\left[\frac{n(\bar{x})}{\bar{x}}\left(2 - \frac{4}{\bar{x}^2} + R_1 - R_2\bar{x}^2\right)\right]}{\left[\frac{n(\bar{x})}{\bar{x}}\left(\frac{2}{\bar{x}^2} + R_1 + R_2\bar{x}^2\right)\right]}\right\} \\
&= \frac{\frac{n(\bar{x})}{\bar{x}}\left(2 - \frac{4}{\bar{x}^2} + R_1 - R_2\bar{x}^2\right)}{\frac{n(\bar{x})}{\bar{x}}\left(2 + R_1\bar{x}^2 + R_2\bar{x}^4\right)} \underset{\bar{x}\uparrow+\infty}{\longrightarrow} 1
\end{aligned}$$

where $|R_1| < 3/\bar{x}^4$ and $|R_2| < 15/\bar{x}^6$, by Lemma A1(f).

Therefore, from (57), (58) and (59), when γ_F is very small,

$$\begin{aligned}
(60)\quad \frac{dU^F}{d\gamma_F} &= 2\gamma_F\Bigg[\underbrace{N(\bar{x}) - N(-\bar{x})}_{\to 1} - \underbrace{\frac{\gamma_F^2}{\sigma_F^2}}_{\to 0}\underbrace{n(\bar{x})\bar{x}^2}_{\to 0}\underbrace{\frac{1}{\bar{x}^2}\left(\frac{N(-\bar{x}) + \bar{x}n(\bar{x}) - \bar{x}^2 N(-\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})}\right)}_{\to 1}\Bigg] \\
&\underset{\bar{x}\uparrow+\infty}{\longrightarrow} 2\gamma_F > 0
\end{aligned}$$

The fly's utility is increasing in γ_F when γ_F is close to zero. This proves Theorem 4.2. ■

Appendix E Expertise

E.1 Social welfare and σ_n

In this section, we prove Theorem 5.1, which says that fighting corruption and training the fly are complements.

Proof of Theorem 5.1. Now, we consider what happens to the society in response to the fly's expertise σ_n. From (27), we express social welfare in terms of $\underline{X}$, $\bar{X}$, σ_F, β^I, and σ_S. In the model, σ_S is a primitive exogenous parameter, γ_F, γ_T and σ_n are exogenous social choices for fighting corruption and training the flies. For computing (18), we are varying σ_n but leaving σ_S, γ_T, and γ_F fixed, which also implies the derived coefficient $\sigma_T = \sqrt{\sigma_S^2 + \gamma_T^2}$ is fixed. To use the chain rule to compute $dW^S/d\sigma_n$, we use the following dependencies for variables that are not constant and depend directly or indirectly on σ_n:

(a) W^S depends on V^S and σ_n: see (1).
(b) V^S depends on $\underline{X}$, $\bar{X}$, β^I and σ_F: see (27).
(c) $\underline{X}$, $\bar{X}$ depend on σ_F and β^T: see (25) and definitions $\underline{x} \equiv \underline{X}/\sigma_F$ and $\bar{x} \equiv \bar{X}/\sigma_F$.
(d) β^I depends on σ_n: see the text after (6).
(e) σ_F depends on σ_n: see (9).
(f) β^T depends on σ_n: see (11).

Then, the total derivative of social welfare with respect to σ_n is:

$$\begin{aligned}(61)\quad \frac{dW^S}{d\sigma_n} = {} & \frac{\partial V^S}{\partial \sigma_F}\frac{d\sigma_F}{d\sigma_n} + \frac{\partial V^S}{\partial \beta^I}\frac{d\beta^I}{d\sigma_n} + \frac{\partial V^S}{\partial \underline{X}}\left(\frac{\partial \underline{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\sigma_n} + \frac{\partial \underline{X}}{\partial \beta^T}\frac{d\beta^T}{d\sigma_n}\right) \\ & + \frac{\partial V^S}{\partial \bar{X}}\left(\frac{\partial \bar{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\sigma_n} + \frac{\partial \bar{X}}{\partial \beta^T}\frac{d\beta^T}{d\sigma_n}\right) - \frac{dC_n(\sigma_n)}{d\sigma_n}\end{aligned}$$

where the terms correspond to the terms in (18) but have been expanded more.

I. Fixed $\underline{X}$ and $\bar{X}$

Now, let's fix $\underline{X}$ and $\bar{X}$ throughout this section.

$$(62)\quad \frac{\partial V^S}{\partial \beta^I}\Big|_{\sigma_F,\underline{X},\bar{X}} = \int_{\varphi=-\infty}^{\underline{X}/\sigma_F} 2\underline{X}\frac{\sigma_S^2}{\sigma_F}\varphi n(\varphi)\,d\varphi + \int_{\varphi=\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} 2\sigma_S^2\varphi^2 n(\varphi)\,d\varphi + \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} 2\bar{X}\frac{\sigma_S^2}{\sigma_F}\varphi n(\varphi)\,d\varphi$$

$$= -2\underline{X}\frac{\sigma_S^2}{\sigma_F}n(\varphi)\Big|_{-\infty}^{\underline{X}/\sigma_F} + 2\sigma_S^2\int_{\varphi=\underline{X}/\sigma_F}^{\bar{X}/\sigma_F}[n''(\varphi)+n(\varphi)]\,d\varphi - 2\bar{X}\frac{\sigma_S^2}{\sigma_F}n(\varphi)\Big|_{\bar{X}/\sigma_F}^{+\infty}$$

$$= -2\underline{X}\frac{\sigma_S^2}{\sigma_F}n(\underline{X}/\sigma_F) + 2\sigma_S^2\big[-\varphi n(\varphi)\big|_{\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} + N(\varphi)\big|_{\underline{X}/\sigma_F}^{\bar{X}/\sigma_F}\big] + 2\bar{X}\frac{\sigma_S^2}{\sigma_F}n(\bar{X}/\sigma_F)$$

$$= -2\underline{x}\sigma_S^2 n(\underline{x}) - 2\sigma_S^2\bar{x}n(\bar{x}) + 2\sigma_S^2\underline{x}n(\underline{x}) + 2\sigma_S^2[N(\bar{x})-N(\underline{x})] + 2\bar{x}\sigma_S^2 n(\bar{x})$$

$$= 2\sigma_S^2[N(\bar{x})-N(\underline{x})]$$

Therefore, from Equations (30), (69), (61), (70) and (62)

$$(63)\quad \frac{\partial V^S}{\partial \sigma_n}\Big|_{\underline{X},\bar{X}} = \frac{\partial V^S}{\partial \sigma_F}\frac{d\sigma_F}{d\sigma_n} + \frac{\partial V^S}{\partial \beta^I}\frac{d\beta^I}{d\sigma_n}$$

$$= \Big(2\sigma_F(\beta^S-1)[\underline{x}n(\underline{x})-\bar{x}n(\bar{x})] - 2\sigma_F[N(\bar{x})-N(\underline{x})]\Big)\Big(-\frac{\sigma_n\sigma_T^4}{\sigma_F\sigma_I^4}\Big)$$

$$+ 2\sigma_S^2[N(\bar{x})-N(\underline{x})]\Big(-\frac{2\sigma_T^2\sigma_n}{\sigma_I^4}\Big)$$

$$= \frac{4\sigma_n\sigma_T^4}{\sigma_I^4}(\beta^S-1)\bar{x}n(\bar{x}) + \frac{2\sigma_n\sigma_T^4}{\sigma_I^4}[N(\bar{x})-N(\underline{x})] - \frac{4\sigma_n\sigma_S^2\sigma_T^2}{\sigma_I^4}[N(\bar{x})-N(\underline{x})]$$

$$= \frac{4\sigma_n\sigma_T^4}{\sigma_I^4}(\beta^S-1)\bar{x}n(\bar{x}) + \frac{2\sigma_n\sigma_T^2(\sigma_T^2-2\sigma_S^2)}{\sigma_I^4}[N(\bar{x})-N(\underline{x})]$$

When both γ_F and γ_T are small, $\underline{X}\to-\infty$, $\bar{X}\to+\infty$.

$$\frac{\partial V^S}{\partial \sigma_n}\Big|_{\underline{X},\bar{X}} = \frac{4\sigma_n\sigma_T^4}{\sigma_I^4}(\beta^S-1)\underbrace{\bar{x}n(\bar{x})}_{\to 0} + \frac{2\sigma_n\sigma_T^2(\sigma_T^2-2\sigma_S^2)}{\sigma_I^4}[\underbrace{N(\bar{x})-N(-\bar{x})}_{\to 1}]$$

$$\underset{\bar{x}\uparrow+\infty}{\longrightarrow} \frac{2\sigma_n\sigma_T^2(\gamma_T^2-\sigma_S^2)}{\sigma_I^4} < 0$$

Thus, for fixed $\underline{X}$ and $\bar{X}$, when the tiger and the fly both have low available corruptions, society is worse off with the increase in the noise σ_n before cost.

II. Tiger chooses $\underline{X}$ and $\bar{X}$ optimally in response with γ_F

Next, we consider what happens to the social welfare with changes in σ_n when the tiger chooses $\underline{X}$ and $\bar{X}$ in response to γ_F.

$$(64)\quad \frac{\partial V^S}{\partial \bar{X}}\left(\frac{\partial \bar{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\sigma_n} + \frac{\partial \bar{X}}{\partial \beta^T}\frac{d\beta^T}{d\sigma_n}\right)$$
$$= -\frac{2\beta^I\gamma_T^2 n(\bar{x})}{\sigma_F}\left[\bar{x}\left(-\frac{(\beta^I)^2\sigma_n}{\sigma_F}\right) + \frac{\sigma_F n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})}\left(-\frac{2(\beta^I)^2\sigma_n\gamma_F^2}{\sigma_F^4}\right)\right]$$
$$= \frac{2(\beta^I)^3\gamma_T^2\sigma_n n(\bar{x})}{\sigma_F^2}\left[\bar{x} + \frac{2\gamma_F^2 n(\bar{x})}{\sigma_F^2(N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x}))}\right]$$

Similarly,

$$(65)\quad \frac{\partial V^S}{\partial \underline{X}}\left(\frac{\partial \underline{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\sigma_n} + \frac{\partial \underline{X}}{\partial \beta^T}\frac{d\beta^T}{d\sigma_n}\right) = \frac{\partial V^S}{\partial \bar{X}}\left(\frac{\partial \bar{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\sigma_n} + \frac{\partial \bar{X}}{\partial \beta^T}\frac{d\beta^T}{d\sigma_n}\right)$$
$$= \frac{2(\beta^I)^3\gamma_T^2\sigma_n n(\bar{x})}{\sigma_F^2}\left[\bar{x} + \frac{2\gamma_F^2 n(\bar{x})}{\sigma_F^2(N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x}))}\right]$$

When both γ_F and γ_T are small enough, $\underline{X} \to -\infty$, $\bar{X} \to +\infty$. From (40) and (57), Equation (61) can be expressed as:

$$(66)\quad \frac{dW^S}{d\sigma_n} = \frac{4\sigma_n\sigma_T^4}{\sigma_I^4}(\beta^S - 1)\bar{x}n(\bar{x}) + \frac{2\sigma_n\sigma_T^2(\gamma_T^2 - \sigma_S^2)}{\sigma_I^4}[N(\bar{x}) - N(\underline{x})]$$
$$+ \frac{4(\beta^I)^3\gamma_T^2\sigma_n n(\bar{x})}{\sigma_F^2}\left[\bar{x} + \frac{2\gamma_F^2 n(\bar{x})}{\sigma_F^2(N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x}))}\right] - C_n'(\sigma_n)$$
$$= \frac{4\sigma_n\sigma_T^4}{\sigma_I^4}\left[(\beta^S - 1)\bar{x}n(\bar{x}) + \frac{\gamma_T^2 - \sigma_S^2}{2\sigma_T^2}[N(\bar{x}) - N(\underline{x})]\right.$$
$$\left. + \frac{\beta^I\gamma_T^2 n(\bar{x})\bar{x}^3}{\sigma_F^2}\left(1/\bar{x}^2 + \frac{2\gamma_F^2 n(\bar{x})}{\sigma_F^2\bar{x}^3[N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})]}\right)\right] - C_n'(\sigma_n)$$
$$\approx \frac{4\sigma_n\sigma_T^4}{\sigma_I^4}\left[(\beta^S - 1)\bar{x}n(\bar{x}) + \frac{\gamma_T^2 - \sigma_S^2}{2\sigma_T^2}[N(\bar{x}) - N(\underline{x})]\right.$$
$$\left. + \frac{\beta^I\gamma_T^2 n(\bar{x})\bar{x}^3}{\sigma_F^2}\left(1/\bar{x}^2 + \frac{2\gamma_F^2 n(\bar{x})}{\sigma_F^2 n(\bar{x})(2 + R_1\bar{x}^2 + R_2\bar{x}^4)}\right)\right] - C_n'(\sigma_n)$$
$$\underset{\bar{x}\uparrow+\infty}{\longrightarrow} \frac{4\sigma_n\sigma_T^4}{\sigma_I^4}\left[(\beta^S - 1)\bar{x}n(\bar{x}) + \frac{\gamma_T^2 - \sigma_S^2}{2\sigma_T^2}[N(\bar{x}) - N(\underline{x})]\right.$$
$$\left. + \frac{\beta^I\gamma_T^2 n(\bar{x})\bar{x}^3}{\sigma_F^2}\left(1/\bar{x}^2 + \frac{\gamma_F^2}{\sigma_F^2}\right)\right] - C_n'(\sigma_n)$$

$$= 4\sigma_n(\beta^I)^2\Bigg[(\beta^S-1)\underbrace{\bar{x}n(\bar{x})}_{\to 0}+\underbrace{\frac{(\gamma_T^2-\sigma_S^2)}{2\sigma_T^2}}_{<0}\underbrace{[N(\bar{x})-N(\underline{x})]}_{\to 1}+\underbrace{\frac{\beta^I\gamma_T^2}{\sigma_F^2}}_{<1}\underbrace{n(\bar{x})\bar{x}}_{\to 0}$$

$$+\underbrace{\frac{\beta^I\gamma_T^2}{\sigma_F^2}}_{<1}\underbrace{\frac{\gamma_F^2}{\sigma_F^2}}_{\to 0}\underbrace{n(\bar{x})\bar{x}^3}_{\to 0}\Bigg]-C_n'(\sigma_n)$$

$$\underset{\bar{x}\uparrow+\infty}{\to}\frac{2\sigma_n(\beta^I)^2}{\sigma_T^2}\underbrace{(\gamma_T^2-\sigma_S^2)}_{<0}-C_n'(\sigma_n)$$

where $|R_1|<3/\bar{x}^4$ and $|R_2|<15/\bar{x}^6$, by Lemma A1(f). Thus, when available corruptions are small for the tiger and the fly, society is worse off with the increase in the noise σ_n before cost. This proves Theorem 5.1. ■

E.2 Tiger's utility and σ_n

In this section, we prove Theorem 5.2, which says training the fly generates direct and indirect benefits to the tiger at no cost.

Proof of Theorem 5.2. Now, we consider what happens to the tiger in response to the fly's expertise σ_n. From (42), we express the tiger's utility in terms of $\underline{X}$, $\bar{X}$, σ_F, β^I, σ_T and γ_T. In the model, γ_F, γ_T and σ_n are exogenous social choices for fighting corruption and training the flies. For computing (19), we are varying σ_n but leaving σ_S, γ_T, and γ_F fixed, which also implies the derived coefficient $\sigma_T=\sqrt{\sigma_S^2+\gamma_T^2}$ is fixed. To use the chain rule to compute $dU^T/d\sigma_n$, we use the following dependencies for variables that are not constant and depend directly or indirectly on σ_n:

(a) U^T depends on $\underline{X}$, $\bar{X}$, β^I and σ_F: see (49).
(b) $\underline{X}$, $\bar{X}$ depend on σ_F and β^T: see (25) and definitions $\underline{x}\equiv\underline{X}/\sigma_F$ and $\bar{x}\equiv\bar{X}/\sigma_F$.
(c) β^I depends on σ_n: see the text after (6).
(d) σ_F depends on σ_n: see (9).
(e) β^T depends on σ_n: see (11).

Then, the total derivative of the tiger's utility with respect to σ_n is:

$$\text{(67)}\quad \frac{dU^T}{d\sigma_n} = \frac{\partial U^T}{\partial \sigma_F}\frac{d\sigma_F}{d\sigma_n} + \frac{\partial U^T}{\partial \beta^I}\frac{d\beta^I}{d\sigma_n} + \frac{\partial U^T}{\partial \underline{X}}\left(\frac{\partial \underline{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\sigma_n} + \frac{\partial \underline{X}}{\partial \beta^T}\frac{d\beta^T}{d\sigma_n}\right) + \frac{\partial U^T}{\partial \bar{X}}\left(\frac{\partial \bar{X}}{\partial \sigma_F}\frac{d\sigma_F}{d\sigma_n} + \frac{\partial \bar{X}}{\partial \beta^T}\frac{d\beta^T}{d\sigma_n}\right)$$

$$\begin{aligned}\text{(68)}\quad \frac{\partial U^T}{\partial \beta^I}\Big|_{\sigma_F,\underline{X},\bar{X}} &= \int_{\varphi=-\infty}^{\underline{X}/\sigma_F} 2\underline{X}\frac{\sigma_T^2}{\sigma_F}\varphi n(\varphi)\,d\varphi + \int_{\varphi=\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} 2\sigma_T^2\varphi^2 n(\varphi)\,d\varphi + \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} 2\bar{X}\frac{\sigma_T^2}{\sigma_F}\varphi n(\varphi)\,d\varphi \\ &= -2\underline{X}\frac{\sigma_T^2}{\sigma_F}n(\varphi)\big|_{-\infty}^{\underline{X}/\sigma_F} + 2\sigma_T^2\int_{\varphi=\underline{X}/\sigma_F}^{\underline{X}/\sigma_F}[n''(\varphi)+n(\varphi)]\,d\varphi - 2\bar{X}\frac{\sigma_T^2}{\sigma_F}n(\varphi)\big|_{\bar{X}/\sigma_F}^{+\infty} \\ &= -2\underline{X}\frac{\sigma_T^2}{\sigma_F}n(\underline{X}/\sigma_F) + 2\sigma_T^2\big[-\varphi n(\varphi)\big|_{\underline{X}/\sigma_F}^{\bar{X}/\sigma_F} + N(\varphi)\big|_{\underline{X}/\sigma_F}^{\bar{X}/\sigma_F}\big] + 2\bar{X}\frac{\sigma_T^2}{\sigma_F}n(\bar{X}/\sigma_F) \\ &= -2\underline{x}\sigma_T^2 n(\underline{x}) - 2\sigma_T^2\bar{x}n(\bar{x}) + 2\sigma_T^2\underline{x}n(\underline{x}) + 2\sigma_T^2[N(\bar{x}) - N(\underline{x})] + 2\bar{x}\sigma_T^2 n(\bar{x}) \\ &= 2\sigma_T^2[N(\bar{x}) - N(\underline{x})]\end{aligned}$$

Since $\sigma_F^2 = (\sigma_T^2)^2/\sigma_I^2 + \gamma_F^2 = \sigma_T^4/(\sigma_T^2+\sigma_n^2) + \gamma_F^2$, then

$$\begin{aligned}\text{(69)}\quad \frac{d\sigma_F}{d\sigma_n} &= \frac{d}{d\sigma_n}\left([\sigma_T^4/(\sigma_T^2+\sigma_n^2)+\gamma_F^2]^{\frac{1}{2}}\right) = \frac{1}{2\sigma_F}\left(-\frac{\sigma_T^4 2\sigma_n}{(\sigma_T^2+\sigma_n^2)^2}\right) \\ &= -\frac{\sigma_n\sigma_T^4}{\sigma_F\sigma_I^4} = -\frac{(\beta^I)^2\sigma_n}{\sigma_F} < 0\end{aligned}$$

$$\text{(70)}\quad \frac{d\beta^I}{d\sigma_n} = \frac{d}{d\sigma_n}\left(\frac{\sigma_T^2}{\sigma_T^2+\sigma_n^2}\right) = -\frac{2\sigma_T^2\sigma_n}{(\sigma_T^2+\sigma_n^2)^2} = -\frac{2\sigma_T^2\sigma_n}{\sigma_I^4}$$

From Equations (44), (68), (69), and (70)

$$\begin{aligned}\text{(71)}\quad \frac{\partial U^T}{\partial \sigma_n}\Big|_{\underline{X},\bar{X}} &= \frac{\partial U^T}{\partial \sigma_F}\frac{d\sigma_F}{d\sigma_n} + \frac{\partial U^T}{\partial \beta^I}\frac{d\beta^I}{d\sigma_n} \\ &= \left(2\sigma_F(\beta^T-1)[\underline{x}n(\underline{x}) - \bar{x}n(\bar{x})] - 2\sigma_F[N(\bar{x}) - N(\underline{x})]\right)\left(-\frac{\sigma_n\sigma_T^4}{\sigma_F\sigma_I^4}\right) \\ &\quad + \left(2\sigma_T^2[N(\bar{x}) - N(\underline{x})]\right)\left(-\frac{2\sigma_T^2\sigma_n}{\sigma_I^4}\right)\end{aligned}$$

$$= \frac{4\sigma_n\sigma_T^4(\beta^T-1)\bar{x}n(\bar{x})}{\sigma_I^4} + \frac{2\sigma_n\sigma_T^4[N(\bar{x})-N(\underline{x})]}{\sigma_I^4} - \frac{4\sigma_n\sigma_T^4[N(\bar{x})-N(\underline{x})]}{\sigma_I^4}$$
$$= \underbrace{\frac{4\sigma_n\sigma_T^4(\beta^T-1)\bar{x}n(\bar{x})}{\sigma_I^4}}_{<0} - \underbrace{\frac{2\sigma_n\sigma_T^4[N(\bar{x})-N(\underline{x})]}{\sigma_I^4}}_{>0} < 0$$

For fixed $\underline{X}$ and $\bar{X}$, the tiger's utility is decreasing in σ_n. Also, this is true if the tiger chooses $\underline{X}$ and $\bar{X}$, since the tiger will choose $\underline{X}$ and $\bar{X}$ to maximize utility and the pointwise maximum of decreasing functions is decreasing.

$$(72)\quad \frac{d\beta^T}{d\sigma_n} = \frac{d}{d\sigma_n}\left(\frac{\beta^I\sigma_T^2}{\beta^I\sigma_T^2+\gamma_F^2}\right) = \frac{d}{d\sigma_n}\left(\frac{(\sigma_T^2/\sigma_I^2)\sigma_T^2}{(\sigma_T^2/\sigma_I^2)\sigma_T^2+\gamma_F^2}\right)$$
$$= \frac{d}{d\sigma_n}\left(\frac{\sigma_T^4}{\sigma_T^4+\sigma_I^2\gamma_F^2}\right) = \frac{d}{d\sigma_n}\left(\frac{\sigma_T^4}{\sigma_T^4+\sigma_T^2\gamma_F^2+\sigma_n^2\gamma_F^2}\right)$$
$$= -\frac{2\sigma_T^4\sigma_n\gamma_F^2}{(\sigma_T^4+\sigma_T^2\gamma_F^2+\sigma_n^2\gamma_F^2)^2} = -\frac{2\sigma_T^4\sigma_n\gamma_F^2}{\left[(\sigma_T^2+\sigma_n^2)\left(\frac{\sigma_T^4}{\sigma_T^2+\sigma_n^2}+\gamma_F^2\right)\right]^2}$$
$$= -\frac{2\sigma_T^4\sigma_n\gamma_F^2}{\sigma_I^4\sigma_F^4} = -\frac{2(\beta^I)^2\sigma_n\gamma_F^2}{\sigma_F^4}$$

From Equations (33), (34), (69), (72),

$$(73)\quad \frac{d\bar{X}}{d\sigma_n} = \frac{\partial\bar{X}}{\partial\sigma_F}\frac{d\sigma_F}{d\sigma_n} + \frac{\partial\bar{X}}{\partial\beta^T}\frac{d\beta^T}{d\sigma_n}$$
$$= \bar{x}\left(-\frac{(\beta^I)^2\sigma_n}{\sigma_F}\right) + \frac{\sigma_F n(\bar{x})}{N(-\bar{x})-\bar{x}n(\bar{x})+\bar{x}^2N(-\bar{x})}\left(-\frac{2(\beta^I)^2\sigma_n\gamma_F^2}{\sigma_F^4}\right)$$
$$= \Bigg(\underbrace{-\frac{(\beta^I)^2\sigma_n}{\sigma_F}}_{<0}\Bigg)\Bigg(\underbrace{\bar{x} + \frac{2\gamma_F^2 n(\bar{x})/\bar{x}^2}{\sigma_F^2[\underbrace{N(-\bar{x})+n(-\bar{x})/(-\bar{x})+N(-\bar{x})/(-\bar{x})^2}_{g(-\bar{x})}]}}_{>0}\Bigg) < 0$$

The final inequality follows from Lemma A1(c), which defines g(x) and implies $g(x) \in (0, +\infty)$. Similarly,

$$(74) \quad \frac{d\underline{X}}{d\sigma_n} = -\frac{d\bar{X}}{d\sigma_n} > 0$$

Thus, discretion is decreasing in noise σ_n. This proves Theorem 5.2. ■