Online Appendix to
“Tigers and Flies: Conflicts of Interest, Discretion and Expertise in a Hierarchy”

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Appendix C  Both Tigers and Flies

In this section, we prove Theorem 3.1, which says that if the tiger and the fly both have great available corruption, fighting the fly’s corruption alone reduces social welfare.

C.1 Social Welfare and $\gamma_F$

Proof of Theorem 3.1. First, we derive an expression for social welfare. Since $S$ and $F$ are jointly normal, both having mean 0, we can write

$$S = \beta_S F + \eta_S,$$

where $\eta_S$ is independent of $F$, $\beta_S = \text{cov}(S,F)/\text{var}(F) = \beta_I \sigma_S^2/\sigma_F^2$, and $\text{var}(\eta_S) = \sigma_S^2 - (\beta_S^2)\sigma_F^2 = \sigma_S^2 - \beta_S^2 \beta_I \sigma_F^2 = \sigma_S^2(1 - \beta_S \beta_I)$.

Recall from (1) that $W_S = V_S - C_T(\gamma_T) - C_F(\gamma_F) - C_n(\sigma_n)$, where $V_S$ is social welfare before cost, which is

$$V_S = -E[\{X - (\beta_S F + \eta_S)\}^2]$$

$$= -E[\{X - \beta_S F\}^2 - 2\eta_S (X - \beta_S F) + \eta_S^2] = -E[(X - \beta_S F)^2] - \text{var}(\eta_S).$$

Now, $E[\eta_S (X - \beta_S F)] = 0$ because $E[\eta_S] = 0$, the fly’s choice of $X$ is a function of $F$, and $\eta_S$ and $F$ are independent. From (1), (12) and (26), we have the following expression for social welfare ignoring cost:

$$V_S = -E[\{\pi(\sigma_F \varphi, X, \bar{X}) - \beta_S F\}^2] - \text{var}(\eta_S)$$

$$= \int_{\varphi=-\infty}^{\bar{X}/\sigma_F} -(X - \beta_S \sigma_F \varphi)^2 n(\varphi) \, d\varphi + \int_{\varphi=\bar{X}/\sigma_F}^{\infty} -(\sigma_F \varphi - \beta_S \sigma_F \varphi)^2 n(\varphi) \, d\varphi$$

$$+ \int_{\varphi=\bar{X}/\sigma_F}^{\infty} -(\bar{X} - \beta_S \sigma_F \varphi)^2 n(\varphi) \, d\varphi - \text{var}(\eta_S)$$

$$= \int_{\varphi=-\infty}^{\bar{X}/\sigma_F} (-X^2 + 2X \beta_I \sigma_S^2/\sigma_F + (\beta_I^2 \sigma_S^4/\sigma_F^2) - \varphi^2) n(\varphi) \, d\varphi$$
Equation (27) expresses social welfare in terms of $\bar{X}$, $\bar{X}$, $\sigma_F$, $\beta^I$, and $\sigma_S$. In the model, $\sigma_S$ is a primitive exogenous parameter, $\gamma_F$, $\gamma_T$ and $\sigma_n$ are exogenous social choices for fighting corruption and training the flies. For computing (14), we are varying $\gamma_F$ but leaving $\sigma_S$, $\gamma_T$, and $\sigma_n$ fixed, which also implies the derived coefficients $\beta^I = \sigma_t^2 / (\sigma_T^2 + \sigma_n^2)$, $\sigma_T = \sqrt{\sigma_S^2 + \gamma_T^2}$ and $\sigma_t = \sqrt{\sigma_T^2 + \sigma_n^2}$ are fixed. To use the chain rule to compute $dW^S/d\gamma_F$, we use the following dependencies for variables that are not constant and depend directly or indirectly on $\gamma_F$:

(a) $W^S$ depends on $V^S$ and $\gamma_F$: see (1).
(b) $V^S$ depends on $X$, $X$ and $\sigma_F$: see (27).
(c) $X$, $X$ depend on $\sigma_F$ and $\beta^T$: see (25) and definitions $\bar{x} \equiv X/\sigma_F$ and $\bar{x} \equiv X/\sigma_F$.
(d) $\sigma_F$ depends on $\gamma_F$: see (9).
(e) $\beta^T$ depends on $\gamma_F$: see (11).

Then, the total derivative of social welfare with respect to $\gamma_F$ is:

$$\frac{dW^S}{d\gamma_F} = \frac{\partial V^S}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial V^S}{\partial X} \left( \frac{\partial X}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial X}{\partial \beta^T} \frac{d\beta^T}{d\gamma_F} \right) + \frac{\partial V^S}{\partial \bar{X}} \left( \frac{\partial \bar{X}}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial \bar{X}}{\partial \beta^T} \frac{d\beta^T}{d\gamma_F} \right) - \frac{dC_F(\gamma_F)}{d\gamma_F}$$

where the terms correspond to the terms in (14) but have been expanded more.

I. Fixed $X$ and $\bar{X}$
Thus, for now, we are fixing $X$ and $\bar{X}$ throughout this section. Since $\sigma_F^2 = (\sigma_T^2)^2/\sigma_I^2 + \gamma_F^2$, then

\[
(29) \quad \frac{d\sigma_F}{d\gamma_F} = \frac{d}{d\gamma_F}\left(\left[(\sigma_T^2)^2/(\sigma_I^2 + \gamma_F^2)\right]^{\frac{1}{2}}\right) = \frac{\gamma_F}{\sigma_F} > 0
\]

From equations (27) and (28),

\[
(30) \quad \frac{\partial V^S}{\partial \sigma_F} = \int_{\phi=-\infty}^{X/\sigma_F} - \frac{2X}{\sigma_F^2}\beta^2 \sigma_S^2 \phi n(\phi) \, d\phi + \int_{\phi=X/\sigma_F}^{\infty} -2\sigma_F(\phi^2 n(\phi) \, d\phi \\
\quad + \int_{\phi=X/\sigma_F}^{+\infty} \frac{2X}{\sigma_F^2}\beta^2 \sigma_S^2 \phi n(\phi) \, d\phi
\]

\[
= 2X\beta^2 n(\phi)|_{-\infty}^{-X/\sigma_F} - 2\sigma_F(-\phi n(\phi))|_{\phi}^{X/\sigma_F} - 2\sigma_F N(\phi)\big|_{\phi}^{X/\sigma_F} + 2X\beta^2 n(\phi)|_{\phi}^{+\infty}
\]

\[
= 2X\beta^2 n(X/\sigma_F) + 2\sigma_F \left(\frac{X}{\sigma_F} - \frac{N(X/\sigma_F)}{\sigma_F}\right) + 2\sigma_F[X n(X/\sigma_F)]
\]

\[
\quad - 2\sigma_F[X n(X/\sigma_F) - N(X/\sigma_F)] - 2X\beta^2 n(X/\sigma_F)
\]

\[
= 2\sigma_F(\beta^2 - 1)[X n(X/\sigma_F) - \bar{X} n(\bar{X})] - 2\sigma_F[X n(X/\sigma_F) - N(X/\sigma_F)]
\]

\[
= 2\sigma_F(\beta^2 - 1)[\bar{x} n(\bar{x}) - \bar{n}(\bar{x})] - 2\sigma_F[N(\bar{x}) - N(x)]
\]

\[
= 4\sigma_F(1 - \beta^2)\bar{x} n(\bar{x}) - 2\sigma_F[2N(\bar{x}) - 1]
\]

Let $k_S(\bar{x}) \equiv 4\sigma_F(1 - \beta^2)\bar{x} n(\bar{x}) - 2\sigma_F[2N(\bar{x}) - 1]$, when $\bar{x} > 0$

\[
k_S'(\bar{x}) = 2\sigma_F[2(1 - \beta^2)(n(\bar{x}) - \bar{x}^2 n(\bar{x})) - 2n(\bar{x})]
\]

\[
= 2\sigma_F[2n(\bar{x}) - 2\bar{x}^2 n(\bar{x}) - 2\beta^2 n(\bar{x}) + 2\beta^2 \bar{x}^2 n(\bar{x}) - 2n(\bar{x})]
\]

\[
= 2\sigma_F[2\beta^2 n(\bar{x}) + 2(\beta^2 - 1)\bar{x}^2 n(\bar{x})] < 0
\]

Thus, $k_S(\bar{x})$ is decreasing in $\bar{x}$ when $\bar{x} > 0$. When $\bar{x} \to 0$, $k_S(\bar{x}) \to 0$ Thus, for $0 < \bar{x} < +\infty$, $k_S'(\bar{x}) < 0$ implies that $k_S(\bar{x}) < 0$.

When $\gamma_F$ is very large, $\bar{X} \to 0$, $\partial V^S/\partial \sigma_F < 0$. Therefore,

\[
(31) \quad \frac{\partial V^S}{\partial \sigma_F} \bigg|_{\bar{X}, \bar{x}} = 2\gamma_F(\beta^2 - 1)[\bar{x} n(x) - \bar{n}(\bar{x})] - 2\gamma_F[N(\bar{x}) - N(x)] < 0
\]
Thus, when both $X$ and $\bar{X}$ are fixed, reducing the fly’s available corruption ($\gamma_F$) makes the society better off when $\gamma_F$ is very large.

II. Tiger Chooses $\bar{X}$ and $\bar{X}$ Optimally in Response to $\gamma_F$

Now, we consider what happens to social welfare with changes in $\gamma_F$ when the tiger chooses $\bar{X}$ and $\bar{X}$ optimally in response to $\gamma_F$.

$$\frac{\partial V^S}{\partial X}\bigg|_{\sigma_F, X} = \int_{\bar{X}/\sigma_F}^{+\infty} (-2 \bar{X} + 2 \frac{\beta^T \sigma^2_S}{\sigma_F} \varphi) n(\varphi) d\varphi$$

$$= -2 \bar{X} N(-\bar{X}/\sigma_F) + 2 \frac{\beta^T \sigma^2_S}{\sigma_F} n(\bar{X}/\sigma_F)$$

$$= -2 \sigma_F n(\bar{X}/\sigma_F) \left[ \frac{(\bar{X}/\sigma_F) N(-\bar{X}/\sigma_F)}{n(\bar{X}/\sigma_F)} - \frac{\beta^T \sigma^2_S}{\sigma_F^2} \right]$$

$$= -2 \frac{\beta^T \gamma^2_F n(\bar{X}/\sigma_F)}{\sigma_F} = -2 \frac{\beta^T \gamma^2_F n(\bar{x})}{\sigma^2_F}$$

From equation (25) and the definition $\bar{x} \equiv \bar{X}/\sigma_F$,

$$\frac{\partial \beta^T}{\partial \bar{X}}\bigg|_{\sigma_F} = \frac{\partial}{\partial \bar{X}} \left\{ \frac{(\bar{X}/\sigma_F) N(-\bar{X}/\sigma_F)}{n(\bar{X}/\sigma_F)} \right\}$$

$$= \frac{N(-\bar{X}/\sigma_F)}{\sigma_F n(\bar{X}/\sigma_F)} + \frac{-\bar{X} n(\bar{X}/\sigma_F)}{\sigma_F^2 n(\bar{X}/\sigma_F)} + \frac{\bar{X}^2 N(-\bar{X}/\sigma_F)}{\sigma_F^3 n^2(\bar{X}/\sigma_F)}$$

$$= \frac{1}{\sigma_F n(\bar{x})} \left[ N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x}) \right]$$

Thus,

$$\frac{\partial \bar{X}}{\partial \beta^T}\bigg|_{\sigma_F} = \frac{\sigma_F n(\bar{x})}{N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})}$$

When $\beta^T$ is fixed, $\bar{X}/\sigma_F$ is also fixed. Thus, $\partial(\bar{X}/\sigma_F) = 0$, $\partial \log(\bar{X}/\sigma_F) =$
\[ \partial \log(\bar{X}) - \partial \log(\sigma_F) = 0, \partial \bar{X}/\bar{X} = \partial \sigma_F/\sigma_F, \text{ and} \]

\[ \left(34\right) \quad \frac{\partial \bar{X}}{\partial \sigma_F} \bigg|_{\beta^T} = \frac{\bar{X}}{\sigma_F} = \bar{x} \]

\[ \left(35\right) \quad \frac{d\beta^T}{d\gamma_F} = \frac{d}{d\gamma_F} \left( \frac{\beta^T \sigma^2_F}{\beta^T \sigma^2_T + \gamma^2_F} \right) = -\frac{2\gamma_F \beta^T \sigma^2_F}{(\beta^T \sigma^2_T + \gamma^2_F)^2} = -\frac{2\gamma_F \beta^T}{\sigma^2_F} \]

\[ \left(36\right) \quad \frac{\partial V^S}{\partial X} \left( \frac{\partial X \, d\sigma_F}{\partial \sigma_F d\gamma_F} + \frac{\partial X \, d\beta^T}{\partial \beta_T d\gamma_F} \right) = -\frac{2\beta^T \gamma^2_F n(\bar{x})}{\sigma_F} \left[ \frac{\sigma_F n(\bar{x})}{N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \left( -\frac{2\gamma_F \beta^T}{\sigma^2_F} \right) \right] \]

\[ = -\frac{2\beta^T \gamma^2_F n(\bar{x})}{\sigma_F} \left[ \frac{\sigma_F n(\bar{x})}{N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \left( -\frac{2\gamma_F \beta^T}{\sigma^2_F} \right) \right] \]

\[ = \frac{2\beta^T \gamma^2_F n(\bar{x})}{\sigma^2_F} \left[ \frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \right] \]

Similarily,

\[ \left(37\right) \quad \frac{\partial V^S}{\partial X} \left( \frac{\partial X \, d\sigma_F}{\partial \sigma_F d\gamma_F} + \frac{\partial X \, d\beta^T}{\partial \beta_T d\gamma_F} \right) = \frac{\partial V^S}{\partial X} \left( \frac{\partial \bar{X} \, d\sigma_F}{\partial \sigma_F d\gamma_F} + \frac{\partial \bar{X} \, d\beta^T}{\partial \beta_T d\gamma_F} \right) \]

\[ = \frac{2\beta^T \gamma^2_F n(\bar{x})}{\sigma_F} \left[ \frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \right] \]

When both \( \gamma_T \) and \( \gamma_F \) are very large, \( X \to 0, \bar{X} \to 0, \bar{x} \to 0; \beta^S \to 0, \beta^T \to 0, C_F(\gamma_F) \to 0, C'_F(\gamma_F) \to 0. \) Since \( f'(x) \equiv \lim_{\Delta \to 0} \frac{f(x+\Delta)-f(x-\Delta)}{2\Delta} \), we have

\[ \left(38\right) \quad \lim_{x \to 0} \frac{N(\bar{x}) - N(x)}{2\bar{x}} = n(0) \]

\[ \left(39\right) \quad \lim_{\gamma_F \to +\infty} \frac{\gamma^2_F}{\sigma^2_F} = \lim_{\gamma_F \to +\infty} 1 - \beta^T = \lim_{\gamma_F \to +\infty} 1 - \frac{(\bar{X}/\sigma_F)N(-\bar{X}/\sigma_F)}{n(\bar{X}/\sigma_F)} \]
\[ \lim_{\gamma_F \uparrow +\infty} 1 - \frac{(\bar{X}/\sigma_F)(1/2 - n(0)\bar{X}/\sigma_F)}{n(0)} = \lim_{\gamma_F \uparrow +\infty} 1 - \frac{1}{2n(0)} \left( \frac{\bar{X}}{\sigma_F} \right) + \left( \frac{\bar{X}}{\sigma_F} \right)^2 \]

\[ = \lim_{\gamma_F \uparrow +\infty} 1 - \frac{1}{2n(0)} \left( \frac{\bar{X}}{\sigma_F} \right) = 1 - \frac{\bar{x}}{2n(0)} \]

(40) \[ 0 < \frac{\beta T \gamma_T^2}{\sigma^2_F} = \frac{\beta T (\sigma^2_T - \sigma^2_S)}{\beta T \sigma^2_T + \gamma^2_F} < 1 \]

From Equations (31), (38), (39) and (40),

\[ \frac{dW^S}{d\gamma_F} = 2\gamma_F(\beta^S - 1)[\bar{x}n(x) - \bar{x}n(\bar{x})] - 2\gamma_F[N(\bar{x}) - N(x)] - C'_F(\gamma_F) \]

+ \[ 2 \left[ \frac{2\beta T \gamma_F^2 n(\bar{x})}{\sigma^2_F} \right] \left[ \frac{2\beta T n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-x) - \bar{x}} \right] \]

\[ = 4\gamma_F \bar{x} \left[ (1 - \beta^S)n(\bar{x}) - \frac{N(\bar{x}) - N(x)}{2\bar{x}} \right] \]

+ \[ \frac{\beta T \gamma_T^2 n(\bar{x})}{\sigma^2_F} \left( \frac{2}{1 - \bar{x}n(x)/N(-\bar{x}) + \bar{x}^2 - 1} \right) - C'_F(\gamma_F) \]

\[ \approx 4\gamma_F \bar{x} \left[ (1 - \beta^S)n(\bar{x}) - \frac{N(\bar{x}) - N(x)}{2\bar{x}} + \frac{\beta T \gamma_T^2 n(\bar{x})}{\sigma^2_F} \left( \frac{2}{p(-\bar{x}) - 1} \right) \right] - C'_F(\gamma_F) \]

\[ = 4\gamma_F \bar{x} \left[ (1 - \beta^S)n(0) - n(0) + \frac{\beta T \gamma_T^2 n(0)}{\sigma^2_F} \right] - C'_F(\gamma_F) \]

\[ = 4\gamma_F \bar{x} \left[ \bar{x} n(0) + \frac{\beta T \gamma_T^2 n(0)}{\sigma^2_F} \right] - C'_F(\gamma_F) \]

\[ = 4\gamma_F \bar{x} \left[ \bar{x} n(0) + \frac{\beta T \gamma_T^2 n(0)}{\sigma^2_F} \right] - C'_F(\gamma_F) \]

\[ = 4\gamma_F \bar{x} \left[ \beta T \bar{x} + \frac{\beta T \gamma_T^2 n(0)}{\sigma^2_F} \right] - C'_F(\gamma_F) \]

\[ > 0 \quad \text{when } \gamma_T \text{ is large enough} \]

When both \( \gamma_F \) and \( \gamma_T \) are large enough, social welfare is increasing in \( \gamma_F \).

Thus, when the fly and the tiger both have large available corruptions, fighting the fly’s corruption \( \gamma_F \) without fighting the tiger’s corruption \( \gamma_T \) makes
the society worse off. This is because the damage done by the tiger increasing the fly’s discretion (increasing \( \bar{x} \) or reducing \( x \)) more than offsets the direct benefits of reducing the fly’s available corruption given \( \bar{x} \) and \( x \). This proves Theorem 3.1. ■

C.2 Tiger and \( \gamma_F \)

In this section, we prove Theorem 3.2, which says that the tiger always prefers to fight the fly’s corruption more than is socially optimal. It is because the tiger receives more than proportionally the benefits but bears less than proportionally the costs.

**Proof of Theorem 3.2.** First, we derive an expression for the tiger’s utility. Based on equation (22), the tiger’s utility is

\[
U_T = \gamma_T^2 - E[(\pi(\sigma_F \varphi, X, \bar{X}) - \beta^T F)] - \text{var}(\eta_T)
\]

\[
= \gamma_T^2 - \int_{\varphi = -\infty}^{\bar{X}/\sigma_F} (X - \beta^T \sigma_F \varphi)^2 n(\varphi) d\varphi + \int_{\varphi = \bar{X}/\sigma_F}^{X/\sigma_F} (\sigma_F \varphi + \beta^T \sigma_F \varphi)^2 n(\varphi) d\varphi
\]

\[
+ \int_{\varphi = -\infty}^{\bar{X}/\sigma_F} (X + \beta^T \sigma_F \varphi)^2 n(\varphi) d\varphi - \text{var}(\eta_T)
\]

\[
= \gamma_T^2 - \int_{\varphi = -\infty}^{\bar{X}/\sigma_F} \left( -X^2 + 2X \beta^T \sigma_F^2 \varphi - \frac{(\beta^T \sigma_F^2 \varphi^2)}{\sigma_F^2} \right) n(\varphi) d\varphi
\]

\[
+ \int_{\varphi = \bar{X}/\sigma_F}^{X/\sigma_F} \left( -\sigma_F^2 \varphi^2 + 2 \beta^T \sigma_T^2 \varphi^2 - \frac{(\beta^T \sigma_F^2 \varphi^2)}{\sigma_T^2} \right) n(\varphi) d\varphi
\]

\[
+ \int_{\varphi = \bar{X}/\sigma_F}^{\bar{X}/\sigma_F} \left( -\sigma_F^2 \varphi^2 + 2 \beta^T \sigma_T^2 \varphi^2 - \frac{(\beta^T \sigma_F^2 \varphi^2)}{\sigma_T^2} \right) n(\varphi) d\varphi
\]

\[
= \gamma_T^2 - \int_{\varphi = -\infty}^{\bar{X}/\sigma_F} \left( -X^2 + 2X \beta^T \sigma_F^2 \varphi \right) n(\varphi) d\varphi + \int_{\varphi = \bar{X}/\sigma_F}^{X/\sigma_F} \left( -\sigma_F^2 \varphi^2 + 2 \beta^T \sigma_T^2 \varphi^2 \right) n(\varphi) d\varphi
\]

Equation (42) expresses the tiger’s utility in terms of \( X, \bar{X}, \sigma_F, \beta^T, \sigma_T \) and
\(\gamma_T\). In the model, \(\gamma_F, \gamma_T\) and \(\sigma_n\) are exogenous social choices for fighting corruption and training the flies. For computing (15), we are varying \(\gamma_F\) but leaving \(\sigma_S, \gamma_T,\) and \(\sigma_n\) fixed, which also implies the derived coefficients \(\beta^T = \sigma_T^2/(\sigma_T^2 + \sigma_n^2), \sigma_T = \sqrt{\sigma_S^2 + \gamma_T^2}\) and \(\sigma_I = \sqrt{\sigma_T^2 + \sigma_n^2}\) are fixed. To use the chain rule to compute \(dU^T/d\gamma_F\), we use the following dependencies for variables that are not constant and depend directly or indirectly on \(\gamma_F\):

(a) \(U^T\) depends on \(X, \bar{X}\) and \(\sigma_F\): see (42).
(b) \(X, \bar{X}\) depend on \(\sigma_F\) and \(\beta^T\): see (25) and definitions \(x \equiv X/\sigma_F\) and \(\bar{x} \equiv \bar{X}/\sigma_F\).
(c) \(\sigma_F\) depends on \(\gamma_F\): see (9).
(d) \(\beta^T\) depends on \(\gamma_F\): see (11).

Then, the total derivative of the tiger’s utility with respect to \(\gamma_F\) is:

\[
(43) \quad \frac{dU^T}{d\gamma_F} = \frac{\partial U^T}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial U^T}{\partial X} \left( \frac{\partial X}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial X}{\partial \beta^T} \frac{d\beta^T}{d\gamma_F} \right) + \frac{\partial U^T}{\partial \bar{X}} \left( \frac{\partial \bar{X}}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial \bar{X}}{\partial \beta^T} \frac{d\beta^T}{d\gamma_F} \right)
\]

where the terms correspond to the terms in (15) but have been expanded more.

\[
(44) \quad \frac{\partial U^T}{\partial \sigma_F} \bigg|_{X, \bar{X}} = \int_{\varphi = -\infty}^{X/\sigma_F} -\frac{2X}{\sigma_F^2} \beta^T \sigma_T^2 \varphi n(\varphi) \, d\varphi + \int_{\varphi = X/\sigma_F}^{+\infty} -2\sigma_F \varphi^2 n(\varphi) \, d\varphi
\]

\[
= 2X \beta^T n(\varphi) \bigg|_{-\infty}^{X/\sigma_F} - 2\sigma_F N(\varphi) \bigg|_{X/\sigma_F}^{X/\sigma_F} + 2\bar{X} \beta^T n(\varphi) \bigg|_{X/\sigma_F}^{+\infty}
\]

\[
= 2X \beta^T n(X/\sigma_F) + 2\sigma_F \bar{X} n(\bar{X}/\sigma_F) - 2\sigma_F \frac{X}{\sigma_F} n(X/\sigma_F) - 2\sigma_F [\bar{X} - X] n(\bar{X}/\sigma_F)
\]

\[
= 4\sigma_F (1 - \beta^T) \bar{x} n(\bar{x}) - 2\sigma_F [2N(\bar{x}) - 1]
\]

\[k_T(\bar{x})\]
Let \( k_T(\bar{x}) \equiv 4\sigma_F (1 - \beta^T) \bar{x} n(\bar{x}) - 2\sigma_F [2N(\bar{x}) - 1] \), when \( \bar{x} > 0 \)

\[
(45) \quad k'_T(\bar{x}) = 2\sigma_F [2(1 - \beta^T)(n(\bar{x}) - \bar{x}^2 n(\bar{x})) - 2n(\bar{x})] \\
= 2\sigma_F [2n(\bar{x}) - 2\bar{x}^2 n(\bar{x}) - 2\beta^T n(\bar{x}) + 2\beta^T \bar{x}^2 n(\bar{x}) - 2n(\bar{x})] \\
< 0
\]

Thus, \( k_T(\bar{x}) \) is decreasing with \( \bar{x} \) when \( \bar{x} > 0 \). When \( \bar{x} \to 0 \), \( k_T(\bar{x}) \to 0 \). Thus, for \( 0 < \bar{x} < +\infty \), \( k'_T(\bar{x}) < 0 \) implies that \( k_T(\bar{x}) < 0 \). From (29), (44) and (45), we have

\[
(46) \quad \frac{\partial U^T}{\partial \sigma_F} \bigg|_{\bar{X}, \bar{X}} \frac{d\sigma_F}{d\gamma_F} = 2\gamma_F (\beta^T - 1)[\bar{x} n(\bar{x}) - \bar{x} n(\bar{x})] - 2\gamma_F [N(\bar{x}) - N(\bar{x})] < 0
\]

For fixed \( \bar{X} \) and \( \bar{X} \), the tiger’s utility is decreasing in \( \gamma_F \). Also, this is true if the tiger chooses \( \bar{X} \) and \( \bar{X} \), since the tiger will choose \( \bar{X} \) and \( \bar{X} \) to maximize utility and the pointwise maximum of decreasing functions is decreasing.

\[\blacksquare\]

### Appendix D Information and Discretion

#### D.1 \( \bar{X} \) and \( \gamma_F \)

In this section, we prove Theorem 4.1, which says that fighting corruption and imposing stringent rules are substitutes from the tiger’s perspective.

**Proof of Theorem 4.1.** From Equations (13), (29), (33), (34), (35),

\[
(47) \quad \frac{d\bar{X}}{d\gamma_F} = \frac{\partial \bar{X}}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial \bar{X}}{\partial \beta^T} \frac{d\beta^T}{d\gamma_F} \\
= \frac{\bar{x} \gamma_F}{\sigma_F} + \frac{\sigma_F n(\bar{x})}{N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \left( -\frac{2\gamma_F \beta^T}{\sigma_F^2} \right)
\]
\[
\gamma_F \left( \bar{x} - \frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \right) = \gamma_F \left( 1 - \frac{2N(-\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \right)
\]

The final inequality follows from Lemma A1(d), which defines \( p(x) \) and implies \( p(x) \in (0,1) \), so \( \frac{2}{p(-\bar{x})} \in (2, +\infty) \). Similarly,

\[
\frac{d\bar{X}}{d\gamma_F} = -\frac{d\bar{X}}{d\gamma_F} > 0
\]

Thus, discretion is decreasing in the fly’s available corruption \( \gamma_F \). This proves Theorem 4.1.

\section*{D.2 Fly and \( \gamma_F \)}

In this section, we prove Theorem 4.2, which says that fighting the fly’s corruption makes the fly worse off when \( \gamma_F \) is small enough and better off when \( \gamma_F \) is large enough.

\textbf{Proof of Theorem 4.2.} First, we derive an expression for the fly’s utility. Based on (7), the fly’s utility is

\[
U^F = \gamma_F^2 + E[-(\pi(\sigma_F \varphi, X, \bar{X}) - \sigma_F \varphi)^2] - \text{var}(\eta_I)
\]

\[
= \gamma_F^2 + \int_{\varphi=-\infty}^{X/\sigma_F} -(X - \sigma_F \varphi)^2 n(\varphi) d\varphi + \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} -(\bar{X} - \sigma_F \varphi)^2 n(\varphi) d\varphi - \sigma_T^2 (1 - \beta^I)
\]

\[
= \sigma_F^2 - \beta^I \sigma_T^2 + \int_{\varphi=-\infty}^{X/\sigma_F} (-\bar{X}^2 + 2\bar{X} \sigma_F \varphi - \sigma_F^2 \varphi^2) n(\varphi) d\varphi
\]

\[
+ \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} (-\bar{X}^2 + 2\bar{X} \sigma_F \varphi - \sigma_F^2 \varphi^2) n(\varphi) d\varphi - \sigma_T^2 (1 - \beta^I)
\]
\[
\begin{align*}
&= \sigma_F^2 - \sigma_T^2 + \int_{\varphi=-\infty}^{X/\sigma_F} (-X^2 + 2X\sigma_F\varphi - \sigma_F^2\varphi^2)n(\varphi) \, d\varphi \\
&\quad + \int_{\varphi=\bar{X}/\sigma_F}^{+\infty} (-\bar{X}^2 + 2\bar{X}\sigma_F\varphi - \sigma_F^2\varphi^2)n(\varphi) \, d\varphi
\end{align*}
\]

Equation (49) expresses the fly’s utility in terms of \(X\), \(\bar{X}\), \(\sigma_F\), and \(\sigma_T\). In the model, \(\gamma_F\), \(\gamma_T\) and \(\sigma_n\) are exogenous social choices for fighting corruption and training the flies. For computing (17), we are varying \(\gamma_F\) but leaving \(\sigma_S\), \(\gamma_T\), and \(\sigma_n\) fixed, which also implies the derived coefficients \(\beta_T = \sigma_T^2/(\sigma_T^2 + \sigma_n^2)\), \(\sigma_T = \sqrt{\sigma_S^2 + \gamma_T^2}\) and \(\sigma_I = \sqrt{\sigma_T^2 + \sigma_n^2}\) are fixed. To use the chain rule to compute \(dU^F/d\gamma_F\), we use the following dependencies for variables that are not constant and depend directly or indirectly on \(\gamma_F\):

(a) \(U^F\) depends on \(X\), \(\bar{X}\) and \(\sigma_F\): see (49).

(b) \(X\), \(\bar{X}\) depend on \(\sigma_F\) and \(\beta^T\): see (25) and definitions \(\bar{x} \equiv X/\sigma_F\) and \(\bar{\bar{x}} \equiv \bar{X}/\sigma_F\).

(c) \(\sigma_F\) depends on \(\gamma_F\): see (9).

(d) \(\beta^T\) depends on \(\gamma_F\): see (11).

Thus, the total derivative of the fly’s utility with respect to \(\gamma_F\) is:

\[
(50) \quad \frac{dU^F}{d\gamma_F} = \frac{\partial U^F}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial U^F}{\partial X} \left( \frac{\partial X}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial X}{\partial \beta^T} \frac{d\beta^T}{d\gamma_F} \right) + \frac{\partial U^F}{\partial \bar{X}} \left( \frac{\partial \bar{X}}{\partial \sigma_F} \frac{d\sigma_F}{d\gamma_F} + \frac{\partial \bar{X}}{\partial \beta^T} \frac{d\beta^T}{d\gamma_F} \right)
\]

where the terms correspond to the terms in (17) but have been expanded more.

I. Fixed \(X\) and \(\bar{X}\)

For now, we are fixing \(X\) and \(\bar{X}\) throughout this section. From Equations (49),

\[
(51) \quad \frac{\partial U^F}{\partial \sigma_F} \bigg|_{X, \bar{X}} = 2\sigma_F + \left( -\frac{X}{\sigma_F^2} \right) (-X^2 + 2X\sigma_F X/\sigma_F - \sigma_F^2 X^2/\sigma_F^2)n(X/\sigma_F) \\
\quad \quad \quad + \int_{\varphi=-\infty}^{X/\sigma_F} \left( 2X\varphi - 2\sigma_F \varphi^2 \right)n(\varphi) \, d\varphi
\]
From Equations (29), (50), and (51),

\[ \gamma \]

Now we consider what happens when the tiger chooses \( \bar{X} \) optimally in response to \( \gamma_F \).

\[ \frac{\partial U^F}{\partial \sigma_F} |_{\bar{X}, \bar{X}} \frac{d\sigma_F}{d\gamma_F} = 2\sigma_F [N(\bar{x}) - N(x)] \frac{\gamma_F}{\sigma_F} = 2\gamma_F [N(\bar{x}) - N(x)] > 0 \]

Therefore, when \( X \) and \( \bar{X} \) are fixed, the fly’s utility is increasing in \( \gamma_F \).

II. Tiger Chooses \( X \) and \( \bar{X} \) Optimally in Response to \( \gamma_F \)

Now we consider what happens when the tiger chooses \( X \) and \( \bar{X} \) optimally in response to \( \gamma_F \),

\[ \frac{\partial U^F}{\partial X} |_{\sigma_F, \bar{X}} = \frac{-1}{\sigma_F} (-\bar{X}^2 + 2\bar{X}^2 \bar{X}^2 - \bar{X}^2) n(\bar{X}/\sigma_F) + \int_{\phi=\bar{X}/\sigma_F}^{+\infty} (-2\bar{X} + 2\sigma_F \varphi) n(\varphi) \, d\varphi + \int_{\phi=\bar{X}/\sigma_F}^{+\infty} (-2\bar{X} + 2\sigma_F \varphi) n(\varphi) \, d\varphi \]

\[ = -2\bar{X} N(\varphi)|_{\bar{X}/\sigma_F}^{+\infty} - 2\sigma_F n(\varphi)|_{\bar{X}/\sigma_F}^{+\infty} \]

\[ = -2\bar{X} N(-\bar{X}/\sigma_F) + 2\sigma_F n(\bar{X}/\sigma_F) \]

\[ = -2\sigma_F n(\bar{X}/\sigma_F) \left( \frac{\bar{X}/\sigma_F N(-\bar{X}/\sigma_F)}{n(\bar{X}/\sigma_F)} - 1 \right) \]

\[ = -2\sigma_F n(\bar{X}/\sigma_F)(\beta^T - 1) = \frac{2\gamma_F^2 n(\bar{x})}{\sigma_F} \]
and (55), Equation (50) can be expressed as
\[\frac{\partial U^F}{\partial X} \left( \frac{\partial X}{\partial \sigma_F} d\sigma_F + \frac{\partial X}{\partial \beta^T} d\gamma_F \right)\]

\[= 2\gamma_F^2 n(\bar{x}) \left[ \frac{\sigma_F}{\bar{N}} \left( \frac{\gamma_F}{\bar{x}} + \frac{\sigma_F n(\bar{x})}{\bar{N}(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \left( -\frac{2\gamma_F \beta^T}{\sigma_F} \right) \right)\]

\[= 2\gamma_F^3 n(\bar{x}) \left[ \frac{\sigma_F}{\bar{N}} \left( \frac{\gamma_F}{\bar{x}} \frac{2\beta^T n(\bar{x})}{\bar{N}(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \right)\]

\[= -\frac{2\gamma_F^3 n(\bar{x})}{\sigma_F^2} \left[ \frac{2\beta^T n(\bar{x})}{\bar{N}(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} - \bar{x} \right]\]

Similarly,
\[\frac{\partial U^F}{\partial X} \left( \frac{\partial X}{\partial \sigma_F} d\sigma_F + \frac{\partial X}{\partial \beta^T} d\gamma_F \right) = \frac{\partial U^F}{\partial X} \left( \frac{\partial X}{\partial \sigma_F} d\sigma_F + \frac{\partial X}{\partial \beta^T} d\gamma_F \right)\]

\[= -\frac{2\gamma_F^3 n(\bar{x})}{\sigma_F^2} \left[ \frac{2\beta^T n(\bar{x})}{\bar{N}(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} - \bar{x} \right]\]

When \(\gamma_F\) is very large, \(X \to 0, \bar{X} \to 0\). From Equations (39), (52), (54), and (55), Equation (50) can be expressed as
\[\frac{dU^F}{d\gamma_F} = 2\gamma_F \left[ N(\bar{x}) - N(\bar{x}) \right] - \frac{4\gamma_F^3 n(\bar{x})}{\sigma_F^2} \left[ \frac{2\beta^T n(\bar{x})}{\bar{N}(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2 N(-\bar{x})} - \bar{x} \right]\]

\[= 4\bar{x}^2 \gamma_F \left[ \frac{N(\bar{x}) - N(\bar{x})}{2\bar{x}^2} - \frac{\gamma_F^2 n(\bar{x})}{\sigma_F^2} \left( \frac{2}{\bar{x} - \bar{x}n(\bar{x})/N(-\bar{x}) + \bar{x}^2 - 1} \right) \right] \]

\[= 4\bar{x}^2 \gamma_F \left[ \frac{N(\bar{x}) - N(\bar{x})}{2\bar{x}^2} - \frac{\gamma_F^2 n(\bar{x})}{\sigma_F^2} \left( \frac{2}{\bar{x} - \bar{x}n(\bar{x})/N(-\bar{x}) + \bar{x}^2 - 1} \right) \right] \]

\[\approx 4\bar{x}^2 \gamma_F \left[ \frac{n(0)}{\bar{x}} - \left( 1 - \frac{1}{2n(0)} \right) \frac{n(0)}{\bar{x}} \left( \frac{2}{\bar{x} - \bar{x}n(\bar{x})/N(-\bar{x}) + \bar{x}^2 - 1} \right) \right] \]

\[= 4\bar{x}^2 \gamma_F \left[ -4n(0)^2 + \frac{1}{2} + 2n(0)\bar{x} \right] < 0 \]

Therefore, the fly’s utility is decreasing in \(\gamma_F\) when \(\gamma_F\) is large.

When \(\gamma_F\) is small, \(X \to -\infty, \bar{X} \to +\infty\) and \(\beta^T \to 1\).

\[\lim_{\gamma_F \to 0} \frac{\gamma_F^2}{\sigma_F^2} = \lim_{\gamma_F \to 0} \frac{\gamma_F^2}{\beta^T \sigma_F^2 + \gamma_F^2} = \lim_{\gamma_F \to 0} \frac{1}{\beta^T \sigma_F^2 + \gamma_F^2} = 0\]

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Equation (50) can be written as

\begin{equation}
\frac{dU^F}{d\gamma_F} = 2\gamma_F \left[ N(\bar{x}) - N(-\bar{x}) - \frac{\gamma_F^2}{\sigma_F^2} n(\bar{x}) \left( \frac{2\beta^T n(\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2N(-\bar{x})} - \bar{x} \right) \right]
\end{equation}

= 2\gamma_F \left[ N(\bar{x}) - N(-\bar{x}) - \frac{\gamma_F^2}{\sigma_F^2} n(\bar{x}) \left( \frac{2\bar{x}N(-\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2N(-\bar{x})} - \bar{x} \right) \right]

= 2\gamma_F \left[ N(\bar{x}) - N(-\bar{x}) - \frac{\gamma_F^2}{\sigma_F^2} n(\bar{x})\bar{x}^2 \frac{1}{\bar{x}^2} \left( \frac{N(-\bar{x}) + \bar{x}n(\bar{x}) - \bar{x}^2N(-\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2N(-\bar{x})} \right) \right]

As a preliminary, let’s consider the expression:

\begin{equation}
\frac{1}{\bar{x}^2} \frac{N(-\bar{x}) + \bar{x}n(\bar{x}) - \bar{x}^2N(-\bar{x})}{N(-\bar{x}) - \bar{x}n(\bar{x}) + \bar{x}^2N(-\bar{x})}
\end{equation}

\begin{align*}
&= \frac{1}{\bar{x}^2} \left\{ \frac{n(\bar{x})}{\bar{x}} \left[ (1 - \frac{1}{\bar{x}^2} + R_1) \right] + \bar{x}n(\bar{x}) - \bar{x}^2 \left[ \frac{n(\bar{x})}{\bar{x}} \left( 1 - \frac{1}{\bar{x}^2} + \frac{3}{\bar{x}^4} + R_2 \right) \right] \right\} \\
&= \frac{1}{\bar{x}^2} \left\{ \frac{n(\bar{x})}{\bar{x}} \left[ 2 - \frac{4}{\bar{x}^2} + R_1 - R_2\bar{x}^2 \right] \right\} \\
&= \frac{n(\bar{x})}{\bar{x}} \left[ \frac{2 - \frac{4}{\bar{x}^2} + R_1 - R_2\bar{x}^2}{2 + R_1\bar{x}^2 + R_2\bar{x}^4} \right] \xrightarrow{\bar{x} \uparrow + \infty} 1
\end{align*}

where \(|R_1| < 3/\bar{x}^4\) and \(|R_2| < 15/\bar{x}^6\), by Lemma A1(f).

Therefore, from (57), (58) and (59), when \(\gamma_F\) is very small,

\begin{equation}
\frac{dU^F}{d\gamma_F} \xrightarrow{\bar{x} \uparrow + \infty} 2\gamma_F > 0
\end{equation}

The fly’s utility is increasing in \(\gamma_F\) when \(\gamma_F\) is close to zero. This proves Theorem 4.2.
Appendix E  Expertise

E.1  Social welfare and $\sigma_n$

In this section, we prove Theorem 5.1, which says that fighting corruption and training the fly are complements.

Proof of Theorem 5.1. Now, we consider what happens to the society in response to the fly’s expertise $\sigma_n$. From (27), we express social welfare in terms of $X$, $\bar{X}$, $\sigma_F$, $\beta^I$, and $\sigma_S$. In the model, $\sigma_S$ is a primitive exogenous parameter, $\gamma_F$, $\gamma_T$ and $\sigma_n$ are exogenous social choices for fighting corruption and training the flies. For computing (18), we are varying $\sigma_n$ but leaving $\sigma_S$, $\gamma_T$, and $\gamma_F$ fixed, which also implies the derived coefficient $\sigma_T = \sqrt{\sigma_S^2 + \gamma_T^2}$ is fixed. To use the chain rule to compute $dW^S/d\sigma_n$, we use the following dependencies for variables that are not constant and depend directly or indirectly on $\sigma_n$:

(a) $W^S$ depends on $V^S$ and $\sigma_n$: see (1).
(b) $V^S$ depends on $X$, $\bar{X}$, $\beta^I$ and $\sigma_F$: see (27).
(c) $X$, $\bar{X}$ depend on $\sigma_F$ and $\beta^T$: see (25) and definitions $\bar{x} \equiv X/\sigma_F$ and $\bar{\bar{x}} \equiv \bar{X}/\sigma_F$.
(d) $\beta^I$ depends on $\sigma_n$: see the text after (6).
(e) $\sigma_F$ depends on $\sigma_n$: see (9).
(f) $\beta^T$ depends on $\sigma_n$: see (11).

Then, the total derivative of social welfare with respect to $\sigma_n$ is:

$$
\frac{dW^S}{d\sigma_n} = \frac{\partial V^S}{\partial \sigma_F} \frac{d\sigma_F}{d\sigma_n} + \frac{\partial V^S}{\partial \beta^I} \frac{d\beta^I}{d\sigma_n} + \frac{\partial V^S}{\partial X} \left( \frac{\partial X}{\partial \sigma_F} \frac{d\sigma_F}{d\sigma_n} + \frac{\partial X}{\partial \beta^T} \frac{d\beta^T}{d\sigma_n} \right)
$$

$$
+ \frac{\partial V^S}{\partial \bar{X}} \left( \frac{\partial \bar{X}}{\partial \sigma_F} \frac{d\sigma_F}{d\sigma_n} + \frac{\partial \bar{X}}{\partial \beta^T} \frac{d\beta^T}{d\sigma_n} \right) - \frac{dC_n(\sigma_n)}{d\sigma_n}
$$

where the terms correspond to the terms in (18) but have been expanded more.

I. Fixed $X$ and $\bar{X}$
Now, let’s fix $X$ and $\bar{X}$ throughout this section.

\begin{align}
(62) \quad \frac{\partial V^S}{\partial \beta I} \bigg|_{\sigma_F, X, \bar{X}} & = \int_{\varphi = -\infty}^{X/\sigma_F} 2X \frac{\sigma_S^2}{\sigma_F} \varphi n(\varphi) d\varphi + \int_{\varphi = X/\sigma_F}^{\bar{X}/\sigma_F} 2\sigma_S^2 \varphi^2 n(\varphi) d\varphi + \int_{\varphi = \bar{X}/\sigma_F}^{+\infty} 2\bar{X} \frac{\sigma_S^2}{\sigma_F} \varphi n(\varphi) d\varphi \\
& = -2X \frac{\sigma_S^2}{\sigma_F} n(\varphi) \bigg|_{X/\sigma_F}^{X/\sigma_F} + 2\sigma_S^2 \int_{\varphi = X/\sigma_F}^{\bar{X}/\sigma_F} \left[n''(\varphi) + n(\varphi)\right] d\varphi - 2\bar{X} \frac{\sigma_S^2}{\sigma_F} n(\varphi) \bigg|_{X/\sigma_F}^{+\infty} \\
& = -2X \frac{\sigma_S^2}{\sigma_F} n(X/\sigma_F) + 2\sigma_S^2 \left[-\varphi n(\varphi) \bigg|_{X/\sigma_F}^{X/\sigma_F} + N(\varphi) \bigg|_{X/\sigma_F}^{\bar{X}/\sigma_F} \right] + 2\bar{X} \frac{\sigma_S^2}{\sigma_F} n(X/\sigma_F) \\
& = -2\bar{x} \sigma_S^2 n(\bar{x}) - 2\sigma_S^2 \bar{x} n(\bar{x}) + 2\sigma_S^2 [N(\bar{x}) - N(\bar{x})] + 2\bar{x} \sigma_S^2 n(\bar{x}) \\
& = 2\sigma_S^2 [N(\bar{x}) - N(\bar{x})]
\end{align}

Therefore, from Equations (30), (69), (61), (70) and (62)

\begin{align}
(63) \quad \frac{\partial V^S}{\partial \sigma_n} \bigg|_{X, \bar{X}} & = \frac{\partial V^S}{\partial \sigma_F} \frac{d\sigma_F}{d\sigma_n} + \frac{\partial V^S}{\partial \beta I} \frac{d\beta I}{d\sigma_n} \\
& = \left(2\sigma_F(\beta^S - 1) [x n(x) - \bar{x} n(\bar{x})] - 2\sigma_F [N(\bar{x}) - N(x)] \right) \left( -\frac{\sigma_n^4}{\sigma_F \sigma_F^4} \right) \\
& \quad + 2\sigma_S^2 [N(\bar{x}) - N(x)] \left( -\frac{2\sigma_S^2 \sigma_n}{\sigma_F^4} \right)
\end{align}

When both $\gamma_F$ and $\gamma_T$ are small, $X \to -\infty$, $\bar{X} \to +\infty$.

\begin{align}
\frac{\partial V^S}{\partial \sigma_n} \bigg|_{X, \bar{X}} & = \frac{4\sigma_n \sigma_T^4 (\beta^S - 1) \bar{x} n(\bar{x})}{\sigma_F^4} + \frac{2\sigma_n \sigma_T^4 (2\sigma_T^2 - 2\sigma_S^2)}{\sigma_F^4} [N(\bar{x}) - N(\bar{x})] \\
& \to_{\bar{x} \to +\infty} \frac{2\sigma_n \sigma_T^2 (\gamma_T^2 - \sigma_S^2)}{\sigma_F^4} < 0
\end{align}

Thus, for fixed $X$ and $\bar{X}$, when the tiger and the fly both have low available corruptions, society is worse off with the increase in the noise $\sigma_n$ before cost.

II. Tiger chooses $X$ and $\bar{X}$ optimally in response with $\gamma_F$
Next, we consider what happens to the social welfare with changes in $\sigma_n$ when the tiger chooses $X$ and $\bar{X}$ in response to $\gamma_F$.

\begin{equation}
\frac{\partial V_S}{\partial X} \left( \frac{\partial X}{\partial \sigma_F} \frac{d\sigma_F}{d\sigma_n} + \frac{\partial X}{\partial \beta^T} \frac{d\beta^T}{d\sigma_n} \right) = -\frac{2\beta^I \gamma_T^2 n(\bar{x})}{\sigma_F} \left[ \bar{x} \left( -\frac{(\beta^I)^2 \sigma_n}{\sigma_F} \right) + \frac{\sigma_F n(\bar{x})}{N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \left( -\frac{2(\beta^I)^2 \sigma_n \gamma_T^2}{\sigma_F^4} \right) \right]
\end{equation}

\begin{equation}
= \frac{2(\beta^I)^3 \gamma_T^2 \sigma_n n(\bar{x})}{\sigma_F^2} \left[ \bar{x} + \frac{2\gamma_T^2 n(\bar{x})}{\sigma_F^2 (N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x}))} \right]
\end{equation}

Similarly,

\begin{equation}
\frac{\partial V_S}{\partial X} \left( \frac{\partial X}{\partial \sigma_F} \frac{d\sigma_F}{d\sigma_n} + \frac{\partial \bar{X}}{\partial \beta^T} \frac{d\beta^T}{d\sigma_n} \right) = \frac{2(\beta^I)^3 \gamma_T^2 \sigma_n n(\bar{x})}{\sigma_F^2} \left[ \bar{x} + \frac{2\gamma_T^2 n(\bar{x})}{\sigma_F^2 (N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x}))} \right]
\end{equation}

When both $\gamma_F$ and $\gamma_T$ are small enough, $X \to -\infty$, $\bar{X} \to +\infty$. From (40) and (57), Equation (61) can be expressed as:

\begin{equation}
\frac{dW_S}{d\sigma_n} = 4 \sigma_n \frac{\sigma_T^4}{\sigma_I^4} (\beta^S - 1) \bar{x} n(\bar{x}) + \frac{2 \sigma_n \sigma_T^2 (\gamma_T^2 - \sigma_T^2)}{\sigma_I^4} \left[ N(\bar{x}) - N(\bar{x}) \right]
+ \frac{4(\beta^I)^3 \gamma_T^2 \sigma_n n(\bar{x})}{\sigma_F^2} \left[ \bar{x} + \frac{2\gamma_T^2 n(\bar{x})}{\sigma_F^2 (N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x}))} \right] - C_n'(\sigma_n)
\end{equation}

\begin{equation}
\approx \frac{4 \sigma_n \sigma_T^4}{\sigma_I^4} \left[ (\beta^S - 1) \bar{x} n(\bar{x}) + \frac{\gamma_T^2 - \sigma_T^2}{2\sigma_T^2} \left[ N(\bar{x}) - N(\bar{x}) \right] \right]
+ \frac{\beta^I \gamma_T^2 \sigma_T n(\bar{x}) \bar{x}^3}{\sigma_T^2} \left( 1/\bar{x}^2 + \frac{2\gamma_T^2 n(\bar{x})}{\sigma_T n(\bar{x)}(2 + R_1 \bar{x}^2 + R_2 \bar{x}^4}) \right) - C_n'(\sigma_n)
\end{equation}

\begin{equation}
\rightarrow_{\bar{x} \uparrow +\infty} \left[ (\beta^S - 1) \bar{x} n(\bar{x}) + \frac{\gamma_T^2 - \sigma_T^2}{2\sigma_T^2} \left[ N(\bar{x}) - N(\bar{x}) \right] \right]
+ \frac{\beta^I \gamma_T^2 \sigma_T n(\bar{x}) \bar{x}^3}{\sigma_T^2} \left( 1/\bar{x}^2 + \frac{\gamma_T^2}{\sigma_T^2} \right) - C_n'(\sigma_n)
\end{equation}
\[
= 4\sigma_n(\beta^I)^2 \left[ (\beta^S - 1)\bar{x}n(\bar{x}) + \frac{(\gamma_T^2 - \sigma_S^2)}{2\sigma_T^2} \left[ N(\bar{x}) - N(\bar{x}) \right] + \frac{\beta^I\gamma_T^2}{\sigma_T^2} n(\bar{x})\bar{x} \right.
\]
\[
+ \frac{\beta^I\gamma_T^2}{\sigma_T^2} \frac{\gamma_F^2}{\sigma_F^2} \frac{n(\bar{x})\bar{x}^3}{<1 \rightarrow 0} - C'_n(\sigma_n)
\]
\[
\rightarrow \frac{2\sigma_n(\beta^I)^2}{\sigma_T^2} \left( \frac{\gamma_T^2 - \sigma_S^2}{<0} \right) - C'_n(\sigma_n)
\]

where \(|R_1| < 3/\bar{x}^4\) and \(|R_2| < 15/\bar{x}^6\), by Lemma A1(f). Thus, when available corruptions are small for the tiger and the fly, society is worse off with the increase in the noise \(\sigma_n\) before cost. This proves Theorem 5.1. ■

E.2 Tiger’s utility and \(\sigma_n\)

In this section, we prove Theorem 5.2, which says training the fly generates direct and indirect benefits to the tiger at no cost.

Proof of Theorem 5.2. Now, we consider what happens to the tiger in response to the fly’s expertise \(\sigma_n\). From (42), we express the tiger’s utility in terms of \(\bar{X}, \bar{X}, \beta^I, \sigma_F\) and \(\gamma_T\). In the model, \(\gamma_F, \gamma_T\) and \(\sigma_n\) are exogenous social choices for fighting corruption and training the flies. For computing (19), we are varying \(\sigma_n\) but leaving \(\sigma_S, \gamma_T,\) and \(\gamma_F\) fixed, which also implies the derived coefficient \(\sigma_T = \sqrt{\sigma_S^2 + \gamma_T^2}\) is fixed. To use the chain rule to compute \(dU_T/d\sigma_n\), we use the following dependencies for variables that are not constant and depend directly or indirectly on \(\sigma_n\):

(a) \(U^T\) depends on \(X, \bar{X}, \beta^I\) and \(\sigma_F\): see (49).
(b) \(X, \bar{X}\) depend on \(\sigma_F\) and \(\beta^I\): see (25) and definitions \(\bar{x} \equiv X/\sigma_F\) and \(\bar{\bar{x}} \equiv X/\sigma_F\).
(c) \(\beta^I\) depends on \(\sigma_n\): see the text after (6).
(d) \(\sigma_F\) depends on \(\sigma_n\): see (9).
(e) \(\beta^T\) depends on \(\sigma_n\): see (11).
Then, the total derivative of the tiger’s utility with respect to $\sigma_n$ is:

\[
(67) \quad \frac{dU^T}{d\sigma_n} = \frac{\partial U^T}{\partial \sigma_F} \frac{d\sigma_F}{d\sigma_n} + \frac{\partial U^T}{\partial \beta^I} \frac{d\beta^I}{d\sigma_n} + \frac{\partial U^T}{\partial X} \left( \frac{\partial X}{\partial \sigma_F} \frac{d\sigma_F}{d\sigma_n} + \frac{\partial X}{\partial \beta^T} \frac{d\beta^T}{d\sigma_n} \right) \\
+ \frac{\partial U^T}{\partial X} \left( \frac{\partial X}{\partial \sigma_F} \frac{d\sigma_F}{d\sigma_n} + \frac{\partial X}{\partial \beta^T} \frac{d\beta^T}{d\sigma_n} \right)
\]

\[
(68) \quad \frac{\partial U^T}{\partial \beta^I} \bigg|_{\sigma_F, \bar{\xi}, \bar{x}} = \int_{\varphi = -\infty}^{X/\sigma_F} 2\frac{\sigma_T^2}{\sigma_F} \varphi n(\varphi) \, d\varphi + \int_{\varphi = X/\sigma_F}^{\infty} 2\sigma_T^2 \varphi^2 n(\varphi) \, d\varphi + \int_{\varphi = X/\sigma_F}^{\infty} 2\varphi_2^2 \varphi n(\varphi) \, d\varphi \\
= -2X \frac{\sigma_T^2}{\sigma_F} n(\varphi) \bigg|_{-\infty}^{X/\sigma_F} + 2\sigma_T^2 \left[ n''(\varphi) + n(\varphi) \right] d\varphi - 2\varphi_2^2 \left( -n(\varphi) \bigg|_{X/\sigma_F} \right) + 2X \frac{\sigma_T^2}{\sigma_F} n(\bar{\xi}/\sigma_F) \\
= -2X \sigma_T^2 n(x) - 2\sigma_T^2 \bar{x} n(x) + 2\sigma_T^2 [N(\bar{x}) - N(x)] + 2\bar{x} \sigma_T^2 n(\bar{x}) \\
= 2\sigma_T^2 [N(\bar{x}) - N(x)]
\]

Since $\sigma_T^2 = (\sigma_T^2)^2/\sigma_F^2 + \gamma_T^2 = \sigma_T^4/(\sigma_T^2 + \sigma_n^2) + \gamma_T^2$, then

\[
(69) \quad \frac{d\sigma_F}{d\sigma_n} = \frac{d}{d\sigma_n} \left[ \frac{\sigma_T^4}{(\sigma_T^2 + \sigma_n^2)} \right] = \frac{1}{2\sigma_T^4} \left( -\sigma_T^2 \frac{2\sigma_n}{(\sigma_T^2 + \sigma_n^2)} \right) \\
= -\frac{\sigma_T^4}{\sigma_F \sigma_T^4} = -\frac{(\beta^I)^2 \sigma_n}{\sigma_T^4} < 0
\]

\[
(70) \quad \frac{d\beta^I}{d\sigma_n} = \frac{d}{d\sigma_n} \left( \frac{\sigma_T^2}{\sigma_T^2 + \sigma_n^2} \right) = -\frac{2\sigma_T^2 \sigma_n}{(\sigma_T^2 + \sigma_n^2)} = -\frac{2\sigma_T^2 \sigma_n}{\sigma_T^4}
\]

From Equations (44), (68), (69), and (70)

\[
(71) \quad \frac{\partial U^T}{\partial \sigma_n} \bigg|_{\bar{\xi}, \bar{x}} = \frac{\partial U^T}{\partial \sigma_F} \frac{d\sigma_F}{d\sigma_n} + \frac{\partial U^T}{\partial \beta^T} \frac{d\beta^T}{d\sigma_n} \\
= \left( 2\sigma_F (\beta^T - 1)(x n(x) - \bar{x} n(\bar{x})) - 2\sigma_F [N(\bar{x}) - N(x)] \right) \left( -\frac{\sigma_T^4}{\sigma_T^2 \sigma_T^4} \right) \\
+ \left( 2\sigma_T^2 [N(\bar{x}) - N(x)] \right) \left( -\frac{2\sigma_T^2 \sigma_n}{\sigma_T^4} \right)
\]

19
\[
\frac{d\beta^T}{d\sigma_n} = \frac{d}{d\sigma_n} \left( \frac{\beta^I \sigma^2_T}{\sigma^2_T + \sigma^2_F + \gamma^2_F} \right) = \frac{d}{d\sigma_n} \left( \frac{(\sigma^2_T/\sigma^2_F)\sigma^2_T}{\sigma^2_T + \sigma^2_F + \gamma^2_F} \right) = \frac{d}{d\sigma_n} \left( \frac{\sigma^4_T}{\sigma^2_T + \sigma^2_F + \sigma^2_n\gamma^2_F + \sigma^2_n\gamma^2_F} \right) = -\frac{2\sigma^4_T\sigma_n\gamma^2_F}{(\sigma^2_T + \sigma^2_F + \sigma^2_n\gamma^2_F)^2} = -\frac{2\sigma^4_T\sigma_n\gamma^2_F}{\sigma^4_F} = -\frac{2(\beta^I)^2\sigma_n\gamma^2_F}{\sigma^4_F}
\]

From Equations (33), (34), (69), (72),

\[
\frac{d\bar{X}}{d\sigma_n} = \frac{\partial \bar{X}}{\partial \sigma_F} \frac{d\sigma_F}{d\sigma_n} + \frac{\partial \bar{X}}{\partial \beta^T} \frac{d\beta^T}{d\sigma_n} = \bar{x} \left( -\frac{(\beta^I)^2\sigma_n}{\sigma_F} \right) + \frac{\sigma_F n(\bar{x})}{\bar{x}^2 N(-\bar{x}) - \bar{x} n(\bar{x}) + \bar{x}^2 N(-\bar{x})} \left( -\frac{2(\beta^I)^2\sigma_n\gamma^2_F}{\sigma^4_F} \right) = \left( -\frac{(\beta^I)^2\sigma_n}{\sigma_F} \right) \left( \bar{x} + \frac{2\gamma^2_F n(\bar{x})/\bar{x}^2}{\sigma_F} \right) < 0
\]

The final inequality follows from Lemma A1(c), which defines \( g(x) \) and im-
plies $g(x) \in (0, +\infty)$. Similarly,

\begin{equation}
\frac{dX}{d\sigma_n} = -\frac{d\bar{X}}{d\sigma_n} > 0
\end{equation}

Thus, discretion is decreasing in noise $\sigma_n$. This proves Theorem 5.2. ■