Problem 1. (i) Suppose that the p.d.f. of a certain random variable $X$ has the following form:

$$f(x) = \begin{cases} cx & 0 < x < 4 \\ 0 & \text{otherwise}, \end{cases}$$

where $c$ is a given constant. Determine the value of $c$ and also the values of $Pr(1 \leq X \leq 2)$ and $Pr(X > 2)$.

(ii) Suppose that a random variable $X$ has a uniform distribution on the interval $[-2, 8]$, find (i) the p.d.f. of $X$; (ii) the value of $Pr(0 < X < 7)$; (iii) the mean and the variance of $X$.

Problem 2 Suppose that the joint p.d.f. of two random variables $X$ and $Y$ is as follows:

$$f(x, y) = \begin{cases} c(x^2 + y) & 0 \leq y \leq 1 - x^2 \\ 0 & \text{otherwise}. \end{cases}$$

Determine (i) the value of the constant $c$; (ii) $Pr(0 \leq X \leq \frac{1}{2})$; (iii) $Pr(Y \leq X + 1)$.

Problem 3 (i) Suppose that $X$ and $Y$ are independent Possion random variables such that $Var(X) + Var(Y) = 5$. Evaluate $P(X + Y < 2)$.

(ii) Suppose that $X_1$ and $X_2$ are independent random variables, and $X_i$ has an exponential distribution with parameter $\beta_i (i = 1, 2)$. Find $Pr(X_1 > kX_2)$, where $k > 0$ is a constant.

Problem 4 Let $X$ and $Y$ have a continuous distribution with joint p.d.f.

$$f(x, y) = \begin{cases} x + y & 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1, \\ 0 & \text{otherwise}. \end{cases}$$

Compute the covariance $Cov(X, Y)$. 