Introduction

Fixed-income securities include bonds whose income is literally fixed, and more generally any claims whose value or risk is related to interest rates and interest rate uncertainty. Quantitative models are particularly useful in this area: fixed-income claims are relatively easy to formulate mathematically, but the subtleties of interest rate dynamics make it so valuation and hedging are not too trivial to figure out.

Some of the players

- Issuers
- Dealers
- Investors
- Pension fund plan sponsors
- Hedgers
- Speculators

Generic issues for the players

- Valuation
- Risk management
- Various sources of risk
  - Interest rate risk
  - Credit risk
  - Foreign exchange (FX) risk
  - Various other
- Tax treatment
- Transaction costs
- Optimal trade-off of risk and return
- Regulatory and contractual constraints
<table>
<thead>
<tr>
<th><strong>Issuers probably need to know</strong></th>
<th><strong>Dealers probably need to know</strong></th>
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<tbody>
<tr>
<td>• How much raising money now will cost them in the future</td>
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<td>• The cheapest way of raising the money</td>
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<td>• What interest risk exposure they are taking on</td>
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<td>• How well the interest risk exposure of assets and liabilities are matched and can be neutralized.</td>
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<td>• Whether funds can be raised more cheaply, for example, by lending at LIBOR plus a spread and swapping into fixed</td>
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<td>• How to hedge the cost of future funding</td>
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<td>• Impact of taxes and transaction costs</td>
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<tr>
<td>• Valuation of the securities, including the impact of credit risk and any embedded options</td>
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<tr>
<td>• How far they are from market-neutral interest rate exposure</td>
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<tr>
<td>• How much interest rate risk they are taking on</td>
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<tr>
<td>• Credit risk exposure</td>
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<td>• Tax status</td>
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<tr>
<td>• Funding sources for inventory</td>
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<tr>
<td>• Liquidity</td>
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<td>• Valuation of the securities</td>
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<td>• If price is away from a fair valuation, when is a correction expected</td>
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<tr>
<td>• Risk and return characteristics of the portfolio</td>
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<td>• Taxes and transaction costs</td>
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<td>• Liquidity</td>
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<td>• What investment will immunize a given pension liability stream</td>
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<td>• Trade-off of risk and returns</td>
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<td>• Impact of transaction costs</td>
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<td>• Liquidity</td>
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<td>• ERISA rules</td>
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Hedgers need to know

- Risk sources and exposures for existing cash flows
- Which risks to hedge and which to absorb/what bets to take
- Risk exposures of various hedging instruments
- Value/transaction cost/tax properties of various hedging instruments

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Some fixed-income securities

- Government issues
  - Federal vs state and local
  - Domestic vs foreign
  - Discount (e.g. T-Bills and STRIPS) vs coupon (T-Bonds and T-Notes)
  - Nominal vs indexed
- Corporate issues
  - Default risk/credit rating important
  - Callable and or convertible?
  - Senior vs junior
- Some interest derivatives
  - T-Bond futures
  - Caps, floors, and collars
  - Inverse floaters

Mortgage-backed securities

- strategic and non-strategic repayment
- various tranches
- i.o.’s and p.o.’s
Risk-return trade-off

Take on risks when:
- they are small so continuing business is not threatened
- taking the risk is rewarded in the market
- you have information that lets you beat the market on average
- taking the risk is your business function

Hedge risks when:
- they are large and threaten survival of your business
- taking the risk is not rewarded
- you are uninformed
- taking the risk is not your business

Valuation
- Market price (most useful for liquid issues)
- Present value and Net present value
  - Must be adapted for nonconstant rates
  - Option pricing gives adjustment for uncertainty
  - Specialized models, e.g., for default risk
- Option pricing models
  - Black-scholes and variants
  - Other “analytic” models
    - Vasicek
    - Cox-Ingersoll-Ross
    - Binomial models
    - Simulation

Hedging
- Duration and Effective Duration
- Analytic option pricing models
- Binomial model
- Simulation

Fixed income risk: important issues
- Riskless depends on maturity
- Real versus nominal
- Hedge cash flows or value
- Tax implications of hedging

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### Transaction costs

- Extra promised return compensates for loss of principal and interest in case of default
- Extra return is approximately the probability of default per unit time times the fraction of the value of a riskless bond that is lost
- Term structure of implied default probabilities

### Taxes

- Break-even tax bracket, for example: $5\%$ yield (T-bill) $\times (1 - 30\% \text{ (tax rate)}) = 3.5\%$ yield (Muni)
- Realizing losses and deferring gains
- Tax timing option

### Illiquid securities

- To first approximation, reduce trading to avoid costs and do not maintain the theoretical hedge
- Illiquid securities (with large costs) should pay a premium and be attractive to long-term investors