Some definitions and formulas

**Spot rate** $r_t$: quoted at $t - 1$ for borrowing/lending from $t - 1$ to $t$

**Forward rate** $f(s, t)$: quoted at $s$ for borrowing/lending from $t - 1$ to $t$

**Discount factor** $D(s, t)$: price at $s$ of receiving 1 at a future date $t$

**Zero-coupon rate** $z(s, t)$: yield at $s$ for a zero-coupon bond maturing at $t$

**Par coupon rate** $c(s, t)$: coupon rate (= yield) quoted at $s$ for a coupon bond maturing at $t$ and trading at par

**Present value** $PV$: value today of a series of future cash flows

**Present value** $NPV$: value today of a series of future cash flows, less the initial price

\[
\begin{align*}
r_t &= f(t - 1, t) \\
D(s, t) &= \frac{1}{\prod_{s=1}^{t-1}(1 + f(0, s))} \\
f(s, t) &= \frac{D(s, t - 1)}{D(s, t)} - 1 \\
D(s, t) &= \frac{1}{(1 + z(s, t))^{t-s}} \\
z(s, t) &= D(s, t)^{-1/(t-s)} - 1 \\
c(s, T) &= \frac{1 - D(s, T)}{\sum_{t=s+1}^{T} D(s, t)} \\
PV &= \sum_{s=1}^{t} D(0, s)c_s \\
NPV &= PV - P \\
&= \sum_{s=0}^{t} D(0, s)c_s
\end{align*}
\]
Zero-coupon and forward rates:

\[ z(0, T) = \left( \prod_{s=1}^{T} (1 + f(0, s)) \right)^{1/T} - 1 \approx \frac{1}{T} \sum_{s=1}^{T} f(0, s) \]

Traditional (Macauley) duration:

\[ duration = \sum_{t=1}^{T} \frac{c_t D(0, t)}{\sum_{s=1}^{T} c_s D(0, s)} t \]

Binomial option pricing:

\[ \text{Value} = R^{-1}(\pi_U V_U + \pi_D V_D) \]
\[ \text{Value} = E^* \left[ \frac{1}{R_1} \frac{1}{R_2} \frac{1}{R_3} \ldots \frac{1}{R_T} C_T \right] \]

For mean reversion

\[ E[r_{t+1} - r_t] = k(\tau - r_t) \]

in the binomial model with equal changes \( \delta \) or \(-\delta\) in rates, set

\[ \pi_U = \frac{1}{2} + \frac{k(\tau - r_t)}{2\delta} \]

Fudge factors:

\[ R_s = R_s^{om} \frac{D(0, s - 1)/D(0, s)}{D^{om}(0, s - 1)/D^{om}(0, s)} \]

or approximately

\[ r_s = r_s^{om} + f(0, s) - f^{om}(0, s) \]