Practice problems for Lecture 5. Answers.

1. Forward and Futures Prices

A forward contact and a futures contract on silver are both one day to maturity. Suppose the futures price is \$7.00/ounce but the forward price is \$6.90/ounce. Assume the spot price tomorrow will be either \$6.85 or \$7.05. Assume futures have cash settlement. Construct an arbitrage.

Arb constructed per ounce

	cash today	cash tomorrow up state	cash tomorrow down state	ounces AG tomorrow
buy AG forw	-	(6.90)	(6.90)	1
sell AG fut	-	7.00-7.05	7.00-6.85	-
sell AG spot	-	7.05	6.85	(1)
net	-	\$0.10	\$0.10	-

yielding a sure arbitrage profit of \$0.10 per ounce tomorrow.

2. Concepts (short answer) Which of the followins situations can be expected to be an arbitrage. Explain briefly why or why not.

a. July wheat futures are 30% more expensive than September wheat futures.

not an arbitrage: It is not possible to short the physical agricultural commodities

b. July wheat futures are 30% cheaper than September wheat futures.

an arbitrage: It should be possible to buy July, store wheat, and sell September.

c. July gold is 30% more expensive than September gold.

an arbitrage: Although it is not normally possible to short physicals, silver and gold are exceptions since existing stocks of the metals are much larger than any anticipated commercial need.

d. July electricity is 30% cheaper than September electricity (at the same location).

not an arbitrage: Storing electricity is very expensive.

3. Futures option pricing (single period)

Riskless bond (interest rate is 20%):

$$100 \longrightarrow 120$$

Futures price:

$$50 \begin{cases} \nearrow 80 \\ \searrow 30 \end{cases}$$

Derivative security (call futures option with strike= 50)

a. What is the portfolio of the futures contract and the bond that replicates the option? (Reminder: you do not put up any money to enter a futures position.)

In a futures contract, you invest 0 at the beginning of the period to receive the change in the futures price at the end of the period, so we have:

$$1 \longrightarrow 1.2$$

$$0 \begin{cases} \nearrow 30 \\ \searrow -20 \end{cases}$$

Let F be the number of futures contracts and let B the number of dollars put in bonds. Then we have

$$30 = 30F + 1.2B$$

$$0 = -20F + 1.2B$$

Then

$$F = 3/5 \quad \text{buy } 3/5 \text{ futures contracts}$$

$$B = 10 \quad \text{buy \$10 worth} = 1/10 \text{ bond}$$

b. What is the price of the replicating portfolio?

Again, keep in mind that it costs 0 to enter a futures position:

$$0F + B = 0 \times 3/5 + 10 = 10$$

c. What are the risk-neutral probabilities of the two states? (Warning: the formula using stock up and down probabilities does not work for futures.)

The risk-neutral probabilities have to price the futures correctly:

$$0 = \frac{\pi_u 30 + (1 - \pi_u)(-20)}{1.2}$$

Therefore,

$$\pi_u = 2/5$$

$$\pi_d = 1 - \pi_u = 3/5$$