Practice Exam (III)

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This is a closed-book examination. Answer all questions as directed. Mark your answers directly on the examination. On the valuation question, make sure your answer is clearly indicated. There are no trick questions on the exam. Good luck!

A. Binomial Option Pricing in One Period *40 points*

Riskless bond (interest rate is 25%):

$$100 \longrightarrow 125$$

Stock:

$$100 \begin{array}{l} \nearrow 245 \\ \searrow 45 \end{array}$$

European put with a strike price of 120:

$$? \begin{array}{l} \nearrow ? \\ \searrow ? \end{array}$$

1. What are the payoffs of the European put in the up and down states?

2. What are the risk-neutral probabilities for the two states tomorrow?

3. What is the price of the European put today?

4. What is the portfolio of stocks and bonds that replicates the put?

B. Concepts (true or false) *20 points*

1. To convert a good deal into a sure thing, we can try to hedge using derivative securities and match cash flows using borrowing and lending.

2. The Black-Scholes model prices American put and call options.

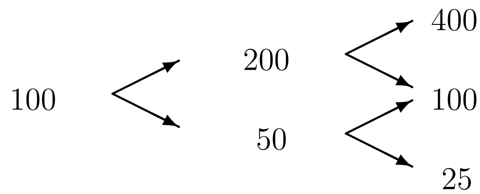
3. American call options on stocks that pay no dividends are often worth more than corresponding European call options.

4. There must be an arbitrage if the 3-month futures price of corn is less than the spot price.

5. Orange County lost a lot of money trading commodity futures options.

C. Binomial Stock Option Pricing *40 points*

The short riskless interest rate is 25% per period. The stock price now is \$100 and will double or fall by half in each period:

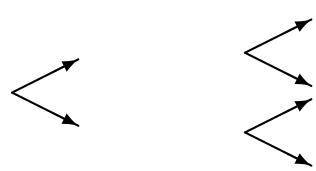


The actual probability of an increase is 60% and the actual probability of a decrease is 40%. Consider European and American options with a strike price of \$75 and maturity two periods from now.

1. What are the risk-neutral probabilities?

2. What are the European put values at each node?

3. What are the American put values at each node?



4. What is the price of a European call option with strike \$75 maturing two periods from now? Use put-call parity.

D. Bonus question *20 points*

Let S_0 be the initial price of a share of stock that pays no dividends. Let r be one plus the riskfree rate, let μ be one plus the mean stock return, and let σ be the standard deviation of the stock return, all per period. What is the price today of a claim maturing 10 periods from now that pays the average stock price over the next 10 periods?

Hint: do not build a binomial tree; this problem requires some insight. For a cash flow T periods away, risk-neutral valuation is $V_0 = E^*[V_T]/r^T$.

Useful Formulas

Binomial model: if the stock has up and down factors u and d and one plus the riskfree rate is r, then the risk-neutral probabilities are

$$\pi_u = \frac{r-d}{u-d}$$

$$\pi_d = \frac{u-r}{u-d}$$

and the one-period option valuation is

$$\text{price} = \frac{1}{r}\left(\pi_u V_u + \pi_d V_d\right).$$

The Black-Scholes call price is

$$C(S,T) = SN(x_1) - BN(x_2),$$

where S is the stock price, $N(\cdot)$ is the cumulative normal distribution function, T is time-to-maturity, B is the bond price $Xe^{-r_f T}$, r_f is the continuously-compounded riskfree rate, σ is the standard deviation of stock returns,

$$x_1 = \frac{\log(S/B)}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T},$$

and

$$x_2 = \frac{\log(S/B)}{\sigma\sqrt{T}} - \frac{1}{2}\sigma\sqrt{T}.$$

Note that $\log(\cdot)$ is the natural logorithm.

The Black-Scholes call price can be approximated by

$$\frac{S-B}{2} + .4\frac{S+B}{2}\sigma\sqrt{T}.$$

The put-call parity formula is

$$B + C = S + P.$$