

## Practice Exam (I) answers

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This is a closed-book examination. Answer all questions as directed. Mark your answers directly on the examination. On the valuation question, make sure your answer is clearly indicated. There are no trick questions on the exam. Good luck!

### A. Binomial Option Pricing in One Period *40 points*

Riskless bond (interest rate is 20%):

100       $\longrightarrow$  120

Stock:

60       $\begin{array}{l} \nearrow 90 \\ \searrow 54 \end{array}$

European put with a strike price of 72:

?       $\begin{array}{l} \nearrow ? \\ \searrow ? \end{array}$

1. What are the payoffs of the European put in the up and down states?

?       $\begin{array}{l} \nearrow 0 \quad \leftarrow \text{up state} \\ \searrow 18 \quad \leftarrow \text{down state} \end{array}$

2. What are the risk-neutral probabilities for the two states tomorrow?

$$\begin{aligned}r &= \frac{120}{100} \\ &= 1.2 \\ u &= \frac{90}{60} \\ &= 1.5 \\ d &= \frac{54}{60} \\ &= 0.9\end{aligned}$$

$$\begin{aligned}\pi_u &= \frac{r - d}{u - d} \\ &= \frac{1.2 - 0.9}{1.5 - 0.9} \\ &= \frac{1}{2} \\ \pi_d &= 1 - \pi_u \\ &= \frac{1}{2}\end{aligned}$$

3. What is the price of the European put today?

$$\begin{aligned}\frac{1}{r}(\pi_u V_u + \pi_d V_d) &= \frac{1}{1.2} \left( \frac{1}{2} 0 + \frac{1}{2} 18 \right) \\ &= 7.5\end{aligned}$$

4. Would the value of the corresponding American put be the same?

No, because an American put pays  $72 - 60 = 12$ , which is more than 7.5, if exercised immediately.

B. Concepts (true or false) *20 points*

1. Absence of arbitrage is the main concept underlying the binomial option pricing model.

true

2. Both puts and calls decline in value when volatility decreases.

true

3. American put options on stocks that pay no dividends are often worth more than corresponding European put options.

true

4. Put-call parity connects the prices of corresponding American puts and calls, the stock, and the bond paying the strike price at maturity.

false (put-call parity is for European options)

5. Stock index futures are a good place to put your money.

false (futures contracts require no investment and are therefore not any place to put your money)

C. Approximate Black-Scholes pricing *40 points*

Consider an at-the-money call option that is two weeks to maturity on a stock with a local standard deviation of 40%/year. Assume the stock is selling for \$50 and the riskfree rate is 5%/year straight interest.

1. What are the variables  $T$ ,  $S$ ,  $B$ , and  $\sigma$  to be used in the option formula?

$$\begin{aligned}T &= 2/52 \approx 0.04 \\S &= 50 \\B &= \frac{50}{1 + rT} \approx 49.9 \\ \sigma &= 0.40\end{aligned}$$

2. What is the call price from the approximate formula?

$$\begin{aligned}C &\approx \frac{S - B}{2} + 0.4 \frac{S + B}{2} \sigma \sqrt{T} \\ &\approx \frac{50 - 49.9}{2} + 0.4 \frac{50 + 49.9}{2} \sigma \sqrt{T} \\ &\approx 1.65\end{aligned}$$

3. What is the corresponding European put price?

By put-call parity,

$$\begin{aligned}P &= B + C - S \\ &\approx 49.9 + 1.65 - 50 \\ &\approx 1.55\end{aligned}$$

D. Bonus question (short answer) *20 points* Answer in no more than three sentences of ordinary length.

Suppose the market interprets news as suggesting that the economy is coming into a period of steady growth. What is the impact of this news on stock index futures? on stock index futures calls? on stock index futures puts?

“Growth” implies a higher stock price and futures price. “Steady growth” implies a lower volatility. Both push the put price down, but the net impact on the call price is ambiguous.