

Some definitions and formulas

spot rate r_t : quoted at $t - 1$ for borrowing/lending from $t - 1$ to t

forward rate $f(s, t)$: quoted at s for borrowing/lending from $t - 1$ to t

discount factor $D(s, t)$: price at s of receiving 1 at a future date t

zero-coupon rate $z(s, t)$: yield at s for a zero-coupon bond maturing at t

par coupon rate $c(s, t)$: coupon rate (= yield) quoted at s for a coupon bond maturing at t and trading at par

present value PV : value today of a series of future cash flows

present value NPV : value today of a series of future cash flows, less the initial price

$$r_t = f(t - 1, t)$$

$$D(s, t) = \frac{1}{\prod_{s=1}^t (1 + f(0, s))}$$

$$f(s, t) = \frac{D(s, t - 1)}{D(s, t)} - 1$$

$$D(s, t) = \frac{1}{(1 + z(s, t))^{t-s}}$$

$$z(s, t) = D(s, t)^{-1/(t-s)} - 1$$

$$c(s, T) = \frac{1 - D(s, T)}{\sum_{t=s+1}^T D(s, t)}$$

$$PV = \sum_{s=1}^t D(0, s)c_s$$

$$\begin{aligned} NPV &= PV - P \\ &= \sum_{s=0}^t D(0, s)c_s \end{aligned}$$

Zero-coupon and forward rates:

$$z(0, T) = \left(\prod_{s=1}^T (1 + f(0, s)) \right)^{1/T} - 1 \approx \frac{1}{T} \sum_{s=1}^T f(0, s)$$

Traditional (Macaulay) duration:

$$duration = \sum_{t=1}^T \frac{c_t D(0, t)}{\sum_{s=1}^T c_s D(0, s)} t$$

binomial option pricing:

$$\text{Value} = R^{-1}(\pi_U V_U + \pi_D V_D)$$

$$\text{Value} = E^* \left[\frac{1}{R_1} \frac{1}{R_2} \frac{1}{R_3} \dots \frac{1}{R_T} C_T \right]$$

For mean reversion

$$E[r_{t+1} - r_t] = k(\bar{r} - r_t)$$

in the binomial model with equal changes δ or $-\delta$ in rates, set

$$\pi_U = \frac{1}{2} + \frac{k(\bar{r} - r_t)}{2\delta}$$

Fudge factors:

$$R_s = R_s^{om} \frac{D(0, s-1)/D(0, s)}{D^{om}(0, s-1)/D^{om}(0, s)}$$

or approximately

$$r_s = r_s^{om} + f(0, s) - f^{om}(0, s)$$