Practice problems for Lecture 3: answers

Note: throughout, make the traditional assumption that the interest rate is positive.

1. (short answer) Answer each question in no more than one sentence of normal length.

a. For call options on a stock that pays no dividends, early exercise is never optimal. However, this is not true in general for put options. Why not?

Delaying exercise of a call gains interest on the strike, but delaying exercise of a put loses interest on the strike.

b. The price of a call option with positive strike price is always less than the stock price. Why?

Getting a share for free now is better than getting a share some of the time later and having to pay for it.

c. The price of a European put option is never greater than the strike price. Why not?

Getting the strike now is better than getting the strike some of the time later and having to give up a share for it.

2. (put-call parity) Sad Corp (SC) is a distressed firm that is not expected to pay dividends over the next year. SC stock is currently at \$10, and it costs \$7 to buy an at-the money call option on SC maturing one year from now. The price of a riskfree zero-coupon bond with a face of \$100 maturing one year from now is \$95. Assume there is no arbitrage.

General: this problem expects you to know that put-call parity applies to European options, and that for non-dividend paying stocks, American and European calls are worth the same but American puts can be worth more than European puts.

a. If the call option described above is a European option, what is the price today of an at-the-money European put option on SC maturing one year from now?

$$S + P = B + C$$

$$P = B + C - S = 9.5 + 7 - 10 = \$6.5$$

b. If the call option described above is an American option, what do we know about the price today of the same European put option?

no change from a, since American and European calls on non-dividend-paying stocks are worth the same

c. If the call option described above is an American option, what do we know about the price today of an at-the-money American put option on SC maturing one year from now?

An American put can be worth more than a European put, so the put value is ≥ 6.5. (More subtle arguments can be used to show that the put value is also $\leq 6.5 + 10 - 9.5 = \$7$. I leave the argument to you.)

3. An investment bank is offering a convertible bond that is 100% backed because it is being offered by a highly-collateralized subsidiary with a AAA rating (so-called triple-A sub). The bond is a 5-year bond paying coupons at an annual rate of 5% per year in the form of two equal semi-annual payments. The bond is convertible into shares of a relatively young and small publicly-traded start-up at the rate of 20 shares of stock (currently worth \$30 per share) per $\$1,000$ of face value of the bond. At the time of exercise of the conversion feature the company pays accrued dividends, namely, the proportion of time between dividends that has passed since the most recent dividend times the size of the next dividend.

a. Show that it is never optimal to exercise the conversion provision before maturity of the bond. (Hint: the argument is similar to the reason why a call option on a non-dividend-paying stock is never exercised early.) Show the cash flows for the dominance argument.

Instead of converting now, we can collect the coupons between now and maturity if we wait to convert. The price variation in the stock and the bond can be replicated by going long in the bond and going short the stock. (We can do even better if we pick and choose when to exercise, but this strategy is good enough to prove that exercising early is dominated.)

The convertible bond is equivalent to a riskless coupon bond plus and option to convert the bond to stock, so it suffices to show that it never pays to exercise this option early. If we exercise the option at maturity $0 \leq t < T = 5$, the option pays value equal to $S_t - B_t$ at time t, i.e.,

	time t	between t and T	time T
exercise option	+share worth S_t -bond worth B_t		

If instead we wait until T to exercise, using a long position in the stock and a short position in the bond to hedge the cash flows, we have cash flows

	time t	between t and T	time T
exercise option			+share worth S_T -bond worth B_T
short stock	+share worth S_t		- share worth S_T
long bond	-bond worth B_t	receive coupons	+bond worth B_T
net flow	+share worth S_t -bond worth B_t	receive coupons	

This dominates exercising now because you receive coupons on top of the other identical flows.

b. What if anything is different if the debt is default-free debt issued by the firm?

In the real world, the debt might sell at a discount because it may not be liquid. However, in an ideal frictionless world the argument should still work.

c. How about if the debt issued by the firm might default?

In that case, it is probably hard to find a comparable bond with a convertible feature to use for the long leg of the dominating position. Using a riskless bond with the same promised cash flows would be worth more than B_t at time t so the payoff of using that bond would be too small at t.