

FIN 524B Exam: answers

Phil Dybvig
May 3, 2016

1. **True-False** (25 points) Make the usual assumption of frictionless markets.

A. Buying a call option is buying the stock and buying volatility.

TRUE

B. When interest rates are negative, it is dominated to exercise an American put option on a stock before maturity.

TRUE (When rates are negative, waiting is beneficial both because you have more information (cherry-picking) and because you receive the strike price later. This is true even if the stock pays dividends, because exercising later means you are giving up the ex-dividend share but not the dividend.)

C. One call option normally has more exposure to the stock price than one share of the underlying stock.

FALSE (The percentage exposure is larger but the absolute exposure is less.)

D. The binomial model is more useful than simulation for valuing mortgage derivatives.

FALSE (Given the many factors affecting mortgage value, a binomial tree would have to be huge.)

E. Considering counterparty risk is usually more important for forward contracts than for futures contracts.

TRUE (You need to worry about counterparty risk for the entire price movement to maturity, not just for one day's variation. Also, the counterparty for a futures is usually an exchange that is backed by many members of the exchange and is unlikely to fail.)

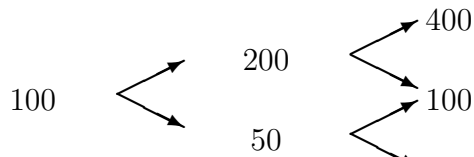
2. **Binomial model: compound option** (30 points) Consider the binomial model with $u = 2$, $d = 1/2$, and $r = 5/4$. The stock price is initially \$100 and the stock is not expected to pay any dividends in the next few periods. The actual probability is $2/3$ for the up state and $1/3$ for the down state.

A. What are the risk-neutral probabilities?

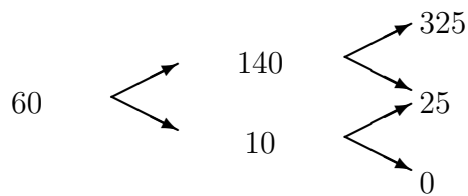
$$\begin{aligned} \pi_u^* &= \frac{r - d}{u - d} \\ &= \frac{5/4 - 1/2}{2 - 1/2} \\ &= \frac{1}{2} \\ \pi_d^* &= 1 - \pi_u^* \\ &= 1/2 \end{aligned}$$

B. Price a European call option with (i) strike price of \$75, (ii) maturity two periods from now, and (iii) underlying is the stock.

stock:

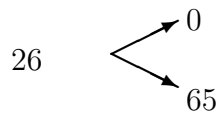


call option:



C. Price a European put option with (i) strike price of \$75, (ii) maturity one period from now, and (iii) underlying is the call option in part B. This is called a *compound option* because it is an option on an option.

put on call:



3. **Put-call Parity** (20 points) A stock with a price of \$75 has two listed options with strike equal to \$88 and a maturity a year from now, a put with a price of \$10 and a call with a price of \$7. The one-year risk-free rate is 10% simple interest. Assume interest rates will always be positive.

A. Show that put-call parity does not hold.

Put-call parity says that $S + P = B + C$ (call + put = bond + call). $B = 88/1.1 = 80$ and the other prices are given as $S = 75$, $P = 10$, and $C = 7$. Therefore, $S + P = 75 + 10 = 85$ but $B + C = 80 + 7 = 87$, so put-call parity does not hold.

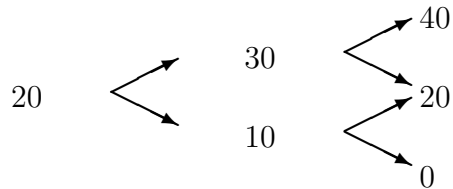
B. Suppose it is known that the stock will not pay any dividends during the year. Could the violation of put-call parity be due to the fact that these are American rather than European options? Explain briefly.

No. Since the interest rate is positive and there are no dividends, the American call price equals the European call price, but the American put price could be larger. Put-call parity should hold for European options, but the American put price being higher could make $S + P > B + C$, but we have the reverse.

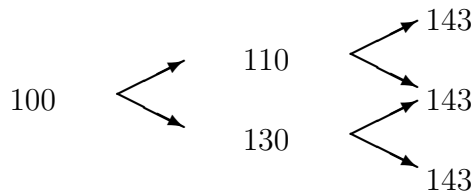
4. **Binomial model: bond option** (30 points) Consider a two-period binomial model in which the short riskless interest rate starts at 20% and moves up or down by 10% each period (i.e., up to 30% or down to 10% at the first change). The artificial probability of each of the two states at any node is 1/2.

A. What is the price at each node of a discount bond with face value of \$143 maturing two periods from the start?

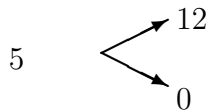
interest (%):



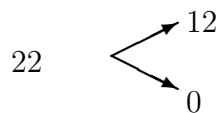
bond price:



B. Price a European put option with (i) strike price of 122, (ii) maturity one period from now, and (iii) underlying is the discount bond in part A.



C. Price an American put option with (i) strike price of 122, (ii) maturity one period from now, and (iii) underlying is the discount bond in part A.



5. **Bonus problem** (20 bonus points) In the setting of problem 2, price an “up-and-in” call option with (i) a strike price of \$10, (ii) two periods to maturity, (iii) underlying is the stock in problem 2, and (iv) a threshold of \$150 the stock price must cross for the option to be activated.

