Portfolio Performance and Agency*

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Abstract

The literature traditionally assumes that a portfolio manager who expends costly effort to generate information makes an unrestricted portfolio choice and is paid according to a sharing rule. However, the revelation principle provides a more efficient institution. If credible communication of the signal is possible, then the optimal contract restricts portfolio choice and pays the manager a fraction of a benchmark plus a bonus proportional to performance relative to the benchmark. If credible communication is not possible, an additional incentive to report extreme signals may be required to remove a possible incentive to underprovide effort and feign a neutral signal.

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The appropriate evaluation and compensation of portfolio managers is an ongoing topic of debate among practitioners and regulators. Although performance measurement and optimal managerial contracting are two sides of the same coin, the academic literature has largely considered the two questions separately. Typically, performance measurement has been studied in the context of models with realistic security returns without consideration of the incentives created by the measure. Optimal contracting has been studied in information models with careful consideration of incentives but simplistic models of portfolio choice and security returns. This paper derives optimal contracts for portfolio managers in the tradition of agency theory¹ but uses a rich model of security returns with full spanning of market states.

This paper's model is not a model of screening managers by ability as in Bhattacharya and Pfleiderer (1985), and in fact the manager's ability is common knowledge from the outset. Rather, there is moral hazard in information production. The manager can expend effort to influence the precision of a private signal about future market prices. The investor's problem is to find a contract for the manager that provides incentives to expend costly effort and to use the signal in the investor's interest while still sharing risk reasonably efficiently. In a similar vein, absence of any information asymmetry at the outset distinguishes this paper from Garcia (2001), whose managers already know their signals at the time of contracting.

Negative results appear to be more common in this literature than positive ones. Stoughton (1993) examines affine (linear plus a constant) and quadratic contracts in a two-asset world. He finds that affine contracts provide no incentives for effort. Quadratic contracts provide some incentive but are not optimal due to their poor risk sharing properties.² Admati and Pfleiderer (1997) contains a similar result to Stoughton's result for affine contracts. There it is shown that contracts that are affine in the excess return over a benchmark also provide no incentives to expend

¹See Ross (1973). For a survey of the agency literature see Laffont and Martimort (2002) or Stole (1993).

²There is a claim in Stoughton (1993) that quadratic contracts approach the first-best in the limit as the investor becomes risk neutral. Sadly, the result is uninterpretable due to an unfortunate choice of utility representation used to define convergence. Using a more reasonable sense of convergence measured by difference in certainty-equivalent, the proof does not work. Intuitively, the problem with using small differences in utilities (instead of say small differences in certainty equivalent) is that the utility representation $U_B(W_B) = -\exp(-bW_B)$ used in that paper becomes very flat as risk aversion *b* falls. For example, $U_B(W_B) \rightarrow -1$ and $U'_B(W_B) \rightarrow 0$ uniformly on bounded sets as $b \downarrow 0$.

costly effort. These negative results arise from an assumption that the contracts do not restrict the portfolio choice of the manager, as would a general contract under the revelation principle.³ Restrictions on the manager's portfolio choice are essential for incentive pay schemes to induce effort. Unrestricted trading may allow a manager to eliminate completely the incentive effects of the fee: whatever leverage is implicit in the fee can be undone in the portfolio choice. Our analysis shows that an optimal contract specifies not only the fee schedule for the agent but also a menu of allowable portfolio strategies.⁴ This form of contract can be motivated by the revelation principle, which implies that this form of contract is general in the sense that it can replicate the equilibrium allocation of any other contract. Actual investment guidelines include many portfolio restrictions, although not necessarily the ones predicted by the model. Common restrictions on asset allocations include restrictions on the universe of assets and ranges for proportions in the various assets; while common restrictions for management within an asset class are limitations on market capitalization or style (growth versus income) of stocks, credit ratings or durations of bonds, restrictions on use of derivatives, maximum allocations to a stock or industry, and increasingly restrictions on portfolio risk measures such as duration, beta, or tracking error.⁵

We derive optimal contracts given a mixture assumption under which the joint density function of the manager's signal and the market state depends affinely on the effort of the manager. We also assume that both the investor and the manager have log utility. In a first-best world, the manager's effort is contractible, and the optimal contract is a proportional sharing rule. In a second-best world, the manager's signal is observable but effort is not contractible, and the optimal fee for the manager is a proportion of the managed portfolio plus a share of the excess return of the portfolio over a benchmark. This gives the appropriate incentive to exert effort. The form of the optimal contract suggests the use of excess return over a benchmark as a measure of portfolio performance, as is common practice

³Gómez and Sharma (2001) have shown that these non-incentive results disappear when a restriction on short-selling is imposed. Similarly, Basak, Pavlova, and Shapiro (2003) show that restricting the deviation from a benchmark can reduce the perverse incentives of an agent facing an *ad hoc* convex objective (motivated by performance-linked future business).

⁴In Admati and Pfeiderer (1997), proposition 5 does examine the effect of adding an affi ne portfolio restriction to the model. However this restriction does not look like an optimal menu, nor does it seem similar to portfolio restrictions observed in practice.

⁵Almazan, Brown, Carlson, and Chapman (2001) documents the prevalence of portfolio restrictions in contracts observed in the mutual fund industry.

in industry. Performance-based fees, when they are observed, are often tied to this sort of measure.

In a third-best world, neither the effort nor the signal is contractible, and additional adjustments are necessary to induce the intended portfolio choice. Relative to the second-best, the third-best contract rewards the manager for reporting more extreme signals, or equivalently, for choosing riskier portfolios. This illustrates the limitations of the second-best contract. With the second-best contract in a third-best world a manager could not fully eliminate the incentives but could mitigate their effect by taking more conservative portfolio positions. The failure of the second-best contract to discourage overly conservative strategies explains the concerns of practitioners about "closet indexers," managers who collect active management fees but adopt passive strategies. In a third-best world the manager's compensation is similar to the second-best but with additional rewards for taking risk.

Conceptually, this paper is very similar to Kihlstrom (1988). However, the model in that paper has only two market states and two signal states, so it does not admit nonaffine contracts. With only two signal states there is also no way for a manager to deviate slightly from the desired investment policy. The only choice is to take the correct position or take the opposite position from what the signal would suggest, and consequently when there are only two signal states the incentive to be overly conservative does not arise. In addition, the investor in the model of Kihlstrom (1988) is risk-neutral. This would imply that no optimal contract exists except that short sales are not allowed. This leads to a corner solution.

Zender (1988) shows that the Jensen measure is the optimal affine contract in a reduced-form model of a mean-variance world. The limitations of that paper are that the mapping from effort to return properties is a black box and that it is unclear what underlying model it is a reduced form for, or indeed whether the optimal contract in the reduced form is also optimal in the underlying model. Palomino and Prat (2003) has a more complex single-period reduced-form model with some unusual assumptions; for example, there is assumed to be an internal maximum of expected return as the risk level varies. Sung (1995)⁶ and Ou-Yang (2003) analyze continuous-time models in which both the drift and diffusion coefficient

⁶The portfolio application is mentioned in Sung (1995) and spelled out in more detail in Sung's thesis, Sung (1991).

can be controlled, and affine contracts arise optimally. As in Zender (1988), the portfolio choice is a reduced form, and it is not clear whether this is the reduced form for a reasonable underlying model.

Finally, a number of economic models are not models of portfolio management but share with our model the feature of having both adverse selection and moral hazard (see Laffont and Martimort (2002), sections 7.1 and 7.2). Very close to our model is the model of delegated expertise of Demski and Sappington (1987), which shares our structure of moral hazard followed by adverse selection. In that model, an analyst exerts costly effort to obtain information. The main differences between that paper and the current paper are that their principal is risk-neutral and the sharing rule over the output is restricted to depend only on output and not on the action taken or the signal observed by the agent. Another literature on the "generalized agency problem," starting with Myerson (1979), has the reverse timing of adverse selection followed by moral hazard. Some recent papers in this literature are Faynzilberg and Kumar (1997), Faynzilberg and Kumar (2000) and Sung (2003).

The paper proceeds as follows. Section I describes the optimal contracting problem. Section II presents analytical solutions in the first-best and second-best cases and discusses the problems that arise in the third-best case. Section III provides numerical examples. Section IV closes the paper.

I The Agency Problem

We consider the contracting problem between an investor and a portfolio manager. This is a moral hazard (or "hidden action") problem because the investor cannot observe the level of costly effort undertaken by the manager. But, it also includes an adverse selection (or "hidden information") problem because the costly effort generates private information that the manager cannot necessarily be trusted to use in the investor's best interest. Our analysis takes the approach of contracting theory and looks for an optimal contract without pre-supposing that the contract conforms to known institutions or has any specific form. The optimal contract derived in this way can be compared with practice or other contracts assumed by other analyses, understanding that an equivalent contract may take a somewhat different appearance.

There are a number of different technologies that can be used to minimize the impact of information problems. For example, information problems can be minimized by using various forms of information gathering before-the-fact, information gathering after-the-fact, and, in a multi-period context, the impact of reputation on future business. Our analysis considers only what can be done using contracting and communication without these other technologies. We pose this in the typical format of an agency problem (as in Ross (1973)), with allowance for a direct mechanism in the signal reporting stage. Here are the assumptions of the model.

Market Returns Investments are made in a market that is complete over states distinguished by security prices. Let $\omega \in \Omega$ denote such a state and let $p(\omega)$ be the pricing density for a claim that pays a dollar in state ω . This is a single-period model in the sense that payoffs will be realized only once, but we think of market completeness as being due to dynamic trading as in a Black-Scholes world. Our agents are "small" and we assume that their trades do not affect market prices. When the state space is not discrete, there may be a technical issue of exactly what space the market is complete over, and we resolve this issue by assuming that a claim is marketed if the integral defining its price exists and is finite.

Information Technology Through costly effort $\varepsilon \in [0, 1]$, the manager has the ability to generate information about the future market state in the form of a private signal $s \in S$. Given effort ε ,

(1)
$$f(s,\omega;\varepsilon) = \varepsilon f^{I}(s,\omega) + (1-\varepsilon)f^{U}(s,\omega).$$

is the probability density of *s* and ω where the market state is ω and the signal is *s*. Here, f^I is an "informed" distribution and f^U is an "uninformed" distribution. We assume that *s* and ω are independent in the uninformed distribution, i.e., $f^U(s, \omega) = f^s(s) f^{\omega}(\omega)$, the product of the marginal distributions. These marginal distributions are assumed to be the same as for the informed distribution. For ω , this must be true or else the manager's effort choice could influence the market return. For *s*, this is a normalization.

One interpretation of the mixture model is that the signal observed by the manager may be informative or it may be uninformative, and the manager cannot tell which. However, the manager knows that expending more effort makes it more likely an informative signal will be generated. Using the mixture model is without loss of generality if there are only two effort levels, and it is a simple sufficient condition for the first-order approach to work in many agency models,⁷ including our second-best problem. Perhaps most importantly, using a mixture model avoids the pathological features of the more common assumption in finance that the agent chooses the precision of a signal joint normally distributed with the outcome; with this common assumption, the unbounded likelihood ratio in the tails makes it too easy to create approximately first-best incentives using a limiting "forcing solution" of Mirrlees (1974). In the mixture model, likelihood ratios are bounded and the Mirrlees forcing solution no longer approaches first-best. As we discuss later in the section, we still have similar results absent the mixture assumption, but the other results are harder to interpret.

Preferences Both the investor and the manager have logarithmic von Neumann-Morgenstern utility of end-of-period consumption, and the manager also bears a utility cost of expending effort. Specifically, the manager's (agent's) utility is $\log(\phi) - c(\varepsilon)$, where ϕ is the manager's fee and $c(\varepsilon)$ is the cost of the effort ε (the hidden action). We shall assume that $c(\varepsilon)$ is differentiable and convex with c'(0) = 0. We will assume that all the problems we consider have optimal solutions.⁸ The investor's (principal's) utility is $\log(V)$, where *V* is the value of what remains in the portfolio after the fee has been paid. Following Grossman and Hart (1983), we will use utility levels rather than consumption levels as the choice variables; this choice makes most of the constraints affine. Our results extend Grossman and Hart (1983) to incorporate a risk-averse principal: this is an important extension for portfolio problems. We will denote by $u_i(s, \omega)$ the investor's equilibrium utility level for only the wealth component $\log(\phi)$ given *s* and ω .

⁷Rogerson (1985) attributes Holmström (1984) with pointing out the appeal of the mixture model as an alternative to the more complex convexity condition of Mirrlees (1976). See also Grossman and Hart (1983) and Hart and Holmström (1987).

⁸For the fi rst-best problem with positive initial wealth, we have in essence a portfolio optimization and an optimal solution exists under growth bounds on the tail probabilities of the state-price density and the asymptotic marginal utility, as in Cox and Huang (1991) or Dybvig, Rogers, and Back (1999). For the other problems, existence can fail in more subtle ways, for example, because compensating the manager enough to induce effort never leaves enough wealth left over to meet the investor's minimum utility level. Or, there may be a closure problem in the second-best like the forcing solution described by Mirrlees (1974) in fi rst-best problems.

Initial Wealth and Reservation Utility The investor's initial wealth is w_0 , and the manager does not have any initial wealth. The agency problem is formulated as maximizing the investor's utility subject to giving the manager a reservation utility level of u_0 . We interpret the reservation utility level as the best the manager can do in alternative employment. In an alternative interpretation, the reservation utility level would be a parameter mapping out the efficient frontier in a bargaining problem between the investor and the manager. Either way, the contracting problem is the same.

Optimal Contracting The contracting problem looks at mechanisms that work in this way. First, there is a contracting phase in which the investor offers a contract to the manager. This contract specifies a portfolio strategy for each possible signal realization and the rule for dividing the portfolio payoff between the investor and the manager. The manager either accepts or refuses the contract; in our formal analysis this is handled as a constraint that says the investor must choose a contract that the manager will be willing to accept. Once the contract is accepted, the manager chooses effort ε and receives the private signal *s*. The manager announces the signal and the portfolio associated with the signal in the contract is selected. Finally, portfolio returns are realized and the manager and the investor divide the portfolio value according to the rule in the contract.

This specification of the problem has a signal announcement that may seem somewhat artificial. This is a *direct mechanism*, which according to the *revelation principle* is guaranteed to duplicate all possible mechanisms, in effect if not in form. Because of the private costly effort, our model does not conform to the traditional derivation of the revelation principle, in which there is private information but no private costly effort. Nonetheless, the revelation principle still works because there are no private actions chosen after the signal is reported (the portfolio choice is reasonably assumed to be public or at least publicly verifiable).⁹ The merit of looking at a direct mechanism is that it permits contracts that implement allocations that can be implemented using the sharing rules traditionally studied in the literature as well as any alternative institutions that may do better. The more general contracts also have a nice economic interpretation in terms of restrictions on the investment strategy.

⁹Laffont and Martimort (2002), section 7.2, discusses how the revelation principle is still valid even with initial costly effort.

The search for an optimal contract is formalized as the solution of a choice problem that makes the investor as well off as possible subject to a budget constraint, the manager's reservation utility level, and incentive-compatibility of the choices we are planning for the manager. We consider three forms of the problem. The first-best assumes that the manager's choice of action and portfolio can be dictated, and as implied by the first theorem of welfare economics it is equivalent to a competitive allocation. The first-best seems unrealistic but it is a useful ideal benchmark and may approximate reality if the agency problem we are concerned about is, for whatever reason, not so important in practice. The second-best requires the manager to want to choose the action optimally but assumes that the use of the information signal in constructing the portfolio can be dictated. This is consistent with an assumption that there is monitoring of the process that ensures the information will be used as intended, or with an assumption that the incentives to misuse the information are handled another way, for example, through loss of business due to a reputation for being a "closet indexer" who collects fees as an active manager but actually chooses a portfolio close to the index. The *third-best* problem has the most profound difficulties with incentives and requires the manager to have the incentive to select the costly effort and also the incentive to reveal truthfully the observed signal. It is an empirical question which is more realistic, the second-best or the third-best.

Choice Problems (original) *First-best:* Choose $u_i(s, \omega)$, $u_m(s, \omega)$, and ε to maximize investor's expected utility,

(2)
$$\iint u_i(s,\omega)(\varepsilon f^I(\omega|s) + (1-\varepsilon)f^{\omega}(\omega))f^s(s)d\omega ds,$$

subject to the budget constraint,

(3)
$$(\forall s \in S) \int (\exp(u_i(s, \omega)) + \exp(u_m(s, \omega))) p(\omega) d\omega = w_0,$$

and the participation constraint,

(4)
$$\iint u_m(s,\omega)(\varepsilon f^I(\omega|s) + (1-\varepsilon)f^{\omega}(\omega))f^s(s)d\omega ds - c(\varepsilon) = u_0.$$

Second-best: add the constraint for the incentive-compatibility of effort:

(5)
$$(\forall \varepsilon' \in \mathcal{E})$$

$$\iint u_m(s, \omega)(\varepsilon f^I(\omega|s) + (1 - \varepsilon)f^{\omega}(\omega))f^s(s)d\omega ds - c(\varepsilon)$$

$$\geq \iint u_m(s, \omega)(\varepsilon' f^I(\omega|s) + (1 - \varepsilon')f^{\omega}(\omega))f^s(s)d\omega ds - c(\varepsilon').$$

Third-best: instead of the constraint (5), add the constraint for simultaneous incentive-compatibility of effort and signal reporting,

(6)
$$(\forall \varepsilon' \in \mathcal{E} \text{ and } \rho : S \to S)$$

$$\iint u_m(s, \omega)(\varepsilon f^I(\omega|s) + (1-\varepsilon)f^{\omega}(\omega))f^s(s)d\omega ds - c(\varepsilon)$$

$$\geq \iint u_m(\rho(s), \omega)(\varepsilon' f^I(\omega|s) + (1-\varepsilon')f^{\omega}(\omega))f^s(s)d\omega ds - c(\varepsilon').$$

In the choice problems, the choice variables are the effort level ε and the utility levels for investor and manager in each contingency (s, ω) . The objective function is expected utility for the investor as computed from the investor's utility level in each contingency and the joint distribution of *s* and ω given the effort level ε . The budget constraint computes the consumptions for investor and manager from their utility levels (the exponential function is the inverse of the logarithmic utility function) and values it using the pricing rule $p(\omega)$. There is a separate budget constraint given each signal realization because there are no opportunities to hedge consumption across signal realizations. The pricing is the same for each *s* because the signal is purely private and because we are making the "small investor" assumption that the manager does not affect pricing in security markets. The participation constraint says that the agent has to be treated well enough to meet the reservation utility level u_0 of outside opportunities.

The integrals used to price payoffs or compute utilities may seem most familiar if the underlying states ω and *s* lie in \Re^n for some *n*. However, the notation and derivations are also consistent with a discrete state space if the measure is a counting measure (implying that the integrals are sums). The notation and derivations are also consistent with more complex state spaces (such as the set of paths of Brownian motion) if integrals are taken with respect to a convenient reference measure.

In the first-best, it is assumed that the effort and the dependence of the portfolio strategy on the signal can be dictated. In the second-best and third-best problems, there are incentive-compatibility conditions that say that the manager has an incentive to choose voluntarily the intended effort ε (second-best and third-best) and reporting the true state *s* (third-best).

We can reduce both the number of choice variables and the number of constraints by use of the following lemma, which enables us to use as the objective the investor's *indirect utility*, which equals the optimal value for the investor given the investor's budget share, the effort level and the realization of the signal.

Lemma 1 In the solution to the original choice problems, the expected utility conditional on s for the investor is given by

(7)
$$\log\left(B_i(s)\frac{f^{\omega}(\omega) + \varepsilon(f^I(\omega|s) - f^{\omega}(\omega))}{p(\omega)}\right)$$

where

(8)
$$B_i(s) = w_0 - \int \exp(u_m(s,\omega))p(\omega)d\omega$$

is the investor's budget share. Therefore, the indirect utility function can be substituted for the original objective in these problems.

PROOF Note that the choice of investor utilities $u_i(s, \omega)$ only appears in the Problems in the objective function (2) and in the budget constraint (3). Therefore, the optimal solution must solve the subproblem of maximizing (2) subject to (3). The first-order condition of this problem is

(9)
$$\left[\varepsilon f^{I}(\omega|s) + (1-\varepsilon)f^{\omega}(\omega)\right]f^{s}(s) = \lambda_{B}(s)p(\omega)\exp(u_{i}(s,\omega))$$

where $\lambda_B(s)$ is the multiplier of the budget constraint. Integrating the above with respect to ω and rearranging gives

$$\lambda_B(s) = \frac{f^s(s)}{B_i(s)}$$

which can be substituted back into the first-order condition to give (7).

Equation (7) can be taken to be an application of the usual formula for optimal consumption given log utility and complete markets (in this case conditional on s). The gross portfolio return

(10)
$$R^P \equiv \frac{\varepsilon f^I(\omega|s) + (1-\varepsilon)f^{\omega}(\omega)}{p(\omega)}$$

is optimal for a log investor conditional on observing *s*.

A related gross portfolio return

(11)
$$R^B \equiv \frac{f^{\omega}(\omega)}{p(\omega)}$$

is optimal for a log investor who does not observe *s*. We will refer to this portfolio as the *benchmark* portfolio, motivated by the fact that benchmark portfolios in practice are intended to be sensible passively-managed portfolios.

Using lemma 1 we can compute the investor's expected utility as

(12)
$$\int \log\left(w_0 - \int \exp(u_m(s,\omega))p(\omega)d\omega\right) f^s(s)ds$$
$$+ \iint \log\left(\frac{\varepsilon f^I(\omega|s) + (1-\varepsilon)f^{\omega}(\omega)}{p(\omega)}\right) (\varepsilon f^I(s,\omega) + (1-\varepsilon)f^{\omega}(\omega)f^s(s))dsd\omega$$

Note that the second term, which we will denote by $K(\varepsilon)$, depends only on effort, ε , and not on the manager's utilities. This means we can ignore this term when solving the problem of what contract will implement a particular effort level and take it into consideration only when optimizing over effort levels. Note also that the first term is concave in the manager's utilities. We will assume $K(\varepsilon)$ is finite for all effort levels ε to avoid some technical difficulties that are far from the main concerns of our paper.

In solving the second-best and third-best problems we desire a more convenient characterization of the incentive-compatibility constraints. We adopt the *first-order approach* of Holmström (1979) to solving principal-agent problems. In the first-order approach, the optimization in each incentive-compatibility condition is replaced by its first-order condition. The manager maximizes

(13)
$$\iint u_m(\rho(s), \omega)(\varepsilon' f^I(\omega|s) + (1-\varepsilon')f^{\omega}(\omega))f^s(s)d\omega ds - c(\varepsilon'),$$

choosing effort ε' in the second-best problem, and choosing both effort ε' and reporting strategy $\rho(s)$ in the second-best problem. We substitute the first-order conditions of this problem, evaluated at $\varepsilon' = \varepsilon$ and $\rho(s) = s$ for the incentive-compatibility constraints, to obtain the first-order problems.

Choice Problems (first-order) First-best: (equivalent to the original first-best)

Choose ε and $u_m(s, \omega)$ to maximize¹⁰

(14)
$$\int \log\left(w_0 - \int \exp(u_m(s,\omega))p(\omega)d\omega\right) f^s(s)ds + K(\varepsilon)$$

subject to manager participation

(15)
$$\iint u_m(s,\omega)(\varepsilon f^I(\omega|s) + (1-\varepsilon)f^{\omega}(\omega))f^s(s)d\omega ds - c(\varepsilon) = u_0$$

Second-best: add the first-order incentive-compatibility of effort choice

(16)
$$\iint u_m(s,\omega)(f^I(\omega|s) - f^{\omega}(\omega))f^s(s)dsd\omega - c'(\varepsilon) = 0.$$

Third-best: in addition to first-order incentive-compatibility of effort choice (16), add the incentive-compatibility of truthful reporting constraint

(17)
$$(\forall s \in S) \int \frac{\partial u_m(s,\omega)}{\partial s} (\varepsilon f^I(\omega|s) + (1-\varepsilon)f^{\omega}(\omega))f^s(s)d\omega = 0.$$

The incentive-compatibility of truthful reporting condition (17) assumes that the support of *s* is a continuum of values so that a derivative is appropriate. If *s* is discrete, then there would be a finite difference condition instead.

II Optimal Contracts

We now describe the solutions to each of the three problems stated above. We begin with the simplest problem, the first-best. Then we demonstrate the impact of the agency problems by showing how the solution changes as we add incentive compatibility constraints in the second-best and third-best.

First-best In a first-best contract we expect to find that there is optimal risk sharing between the manager and the investor. This means that the marginal utility of wealth for the manager should be proportional to the investor's marginal utility in all states.

¹⁰Recall that $K(\varepsilon)$ is the second term of the investor's expected utility function in (12) that doesn't depend on manager utilities.

The first-order condition for u_m is

(18)
$$\frac{\exp(u_m(s,\omega))p(\omega)}{B_i(s)} = \lambda_R(f^{\omega}(\omega) + \varepsilon(f^I(\omega|s) - f^{\omega}(\omega)))$$

where λ_R is the Lagrange multiplier on the participation constraint. Multiplying both sides by $B_i(s)$ and integrating both sides with respect to ω we obtain

(19)
$$B_m(s) = \lambda_R B_i(s).$$

Since the two budget shares must sum to w_0 we have

$$(20) \quad B_i(s) = \frac{w_0}{1 + \lambda_R}$$

from which we obtain

(21)
$$u_m(s,\omega) = \log\left(\frac{w_0\lambda_R}{1+\lambda_R}\frac{f^{\omega}(\omega) + \varepsilon(f^I(\omega|s) - f^{\omega}(\omega))}{p(\omega)}\right).$$

or equivalently the manager's fee is

(22)
$$\phi(s,\omega) = \frac{w_0 \lambda_R}{1 + \lambda_R} R^P,$$

since $u_m(s, \omega) = \log(\phi(s, \omega))$. Comparing this with equation (7), substituting the definition of $B_i(s)$ from above, we see that the first-best contract is a sharing rule that gives the manager a fixed proportion of the payoff of the portfolio independent of the signal. So, as expected, optimal risk sharing obtains. It is worth noting that this result does not depend on the mixture distribution assumption. A proportional sharing rule would still be the first-best contract even under alternative distributional assumptions.

We have not reported the Lagrange multiplier λ_R , but it is easy to do so by substituting the manager's fee (22) into the reservation utility constraint (15).

As mentioned above, the first-best contract assumes that moral hazard and adverse selection are not a problem, and that the effort the manager exerts and the signal the manager observes (not just what is reported) can be contracted upon. However it turns out that even if truthful reporting of the signal cannot be verified the manager will still report truthfully, a result that can be seen as consistent with the notion of preference similarity described by Ross (1974, 1979). In other words, a manager who is constrained to take the first-best effort and is faced with a contract of the form (21) will choose to report the signal honestly, since the budget share does not depend on the reported signal. Misreporting will only affect R^P . But R^P is the gross return on an optimal portfolio for a log investor. Misreporting the signal can only make the manager worse off because it is equivalent to the choice of a suboptimal portfolio.

Connecting the contract in a single-period model with the actual multiperiod economy should not be oversold. However, it is worth observing that this contract resembles the commonly-observed contract paying a fixed proportion of funds under management. Of course, the implications of this contract may be a lot different in our single-period model than in a multiperiod world in which the amount of funds under management can depend on past performance.

Second-best In a second-best world, effort is not observable and therefore the contract must be incentive-compatible for effort.

Proposition 1 The second-best contract gives the manager a payoff that is proportional to the investor's payoff plus a bonus that is proportional to the excess return of the portfolio over the benchmark:

$$\phi(s, \omega) = B_m(R^P + k(R^P - R^B))$$

where B_m and k are non-negative constants.

PROOF We will work with the first-order version of the problem. Here, the first-order condition for $u_m(s, \omega)$ is

(23)
$$\frac{\exp(u_m(s,\omega))p(\omega)}{B_i(s)} = \lambda_R(f^{\omega}(\omega) + \varepsilon(f^I(\omega|s) - f^{\omega}(\omega))) + \lambda_a(f^I(\omega|s) - f^{\omega}(\omega))$$

where λ_a is the Lagrange multiplier on the IC-effort constraint. Proceeding as in the derivation of the first-best case we find that the budget shares are of the same

form as in the first-best contract so that we obtain

(24)
$$u_m(s,\omega) = \log\left(\frac{w_0\lambda_R}{(1+\lambda_R)}\frac{f^{\omega}(\omega) + (\varepsilon + \frac{\lambda_a}{\lambda_R})(f^I(\omega|s) - f^{\omega}(\omega))}{p(\omega)}\right)$$

or equivalently the manager's fee is

(25)
$$\phi(s,\omega) = B_m \left(R^P + k(R^P - R^B) \right)$$

where

(26)
$$k = \frac{\lambda_a}{\epsilon \lambda_R} \ge 0.$$

The difference between this contract and the first-best contract is that the secondbest contract gives the manager a "bonus" that is proportional to the excess return of the fund over a benchmark in addition to a fraction of end-of-period assets under management. This suggests using excess returns over a benchmark as a measure of portfolio performance. This is intriguing since measuring portfolio performance relative to a benchmark is common practice in the portfolio management industry.

The mixture-model assumption plays two roles in this analysis. First, as noted in the literature, it implies that any first-order solution is a solution of the underlying agency model since the first-order conditions for the manager are necessary and sufficient.¹¹ Second, the mixture model assumption implies that the benchmark in the solution can be chosen to be the uninformed optimum.

Absent the mixture model assumption, the optimal contract will include a bonus that is proportional to the excess return over a benchmark but in general this benchmark will not be the uninformed optimum and it may depend on the reported signal. Let $f(\omega|s;\varepsilon)$ be the conditional distribution of the market state given the signal. If this distribution is differentiable in effort and the first-order approach is still valid then the first-order condition for $u_m(s,\omega)$ is

(27)
$$\frac{\exp(u_m(s,\omega))p(\omega)}{B_i(s)} = \lambda_R f(\omega|s;\varepsilon) + \lambda_a f_{\varepsilon}(\omega|s;\varepsilon)$$

¹¹The mixture assumption makes the utility integral affi ne in effort whatever the distributional assumption. Subtracting a convex cost function makes the manager's overall objective concave, implying the fi rst-order conditions are necessary and suffi cient. As we will see later, this does not work in the third-best when there is a signal reporting.

where the subscript indicates partial derivative. When we multiply both sides by state prices and integrate with respect to market states the term involving λ_a drops out because $f(\omega|s;\varepsilon)$ integrates to one for all *s* and we can interchange the order of integration and differentiation. Therefore, the budget share is constant and is of the same form as in the first-best case. The random variable

$$Z \equiv \lambda_a \frac{f_{\varepsilon}(\boldsymbol{\omega}|\boldsymbol{s};\varepsilon)}{p(\boldsymbol{\omega})}$$

is a zero-cost payoff. Because of complete markets this random variable is some excess return. We can interpret this random excess return as the excess return of the managed portfolio over some other portfolio return defined by

$$R^O = R^P - Z.$$

The managers payoff is

$$\phi(s,\omega) = B_m \left[R^P + k' \left(R^P - R^O \right) \right]$$

where $k' = \lambda_a/\lambda_R$. In general of course this "benchmark" R^O will not be the uninformed optimum because it will be some function of *s*, the reported signal (which is okay since the signal is observed in the second-best, but not consistent with the usual choice of a benchmark in practice as an uninformed portfolio).

If the first-order approach fails and there are non-locally-binding incentive compatibility constraints then a similar expression can be derived. The general firstorder condition for the principal's problem may put weights on both local and non-local changes. Combining the weighted average of the corresponding density changes from the optimum and dividing by $p(\omega)$ gives the appropriate change from the optimum to the benchmark. In this general case, the manager still receives a proportion of the portfolio payoff plus a constant times excess return relative a benchmark; however, the benchmark loses the simple interpretation as the uninformed log-optimal portfolio.

Third-best In the first-order third-best, the first-order condition for u_m is

(28)
$$\frac{\exp(u_m(s,\omega))p(\omega)}{B_i(s)} = \lambda_R(f^{\omega}(\omega) + \varepsilon(f^I(\omega|s) - f^{\omega}(\omega))) + \lambda_a(f^I(\omega|s) - f^{\omega}(\omega)) - \varepsilon\lambda_s(s)\frac{\partial f^I(\omega|s)}{\partial s} - \lambda'_s(s)(\varepsilon f^I(\omega|s) + (1-\varepsilon)f^{\omega}(\omega))$$

where $\lambda_s(s)$ is the Lagrange multiplier on the truthful reporting constraint. In this case we have

(29)
$$B_i(s) = \frac{w_0}{1 + \lambda_R - \frac{\lambda'_s(s)}{f^s(s)}}$$

and

(30)
$$B_m(s) = \frac{w_0(\lambda_R - \frac{\lambda'_s(s)}{f^s(s)})}{1 + \lambda_R - \frac{\lambda'_s(s)}{f^s(s)}}.$$

....

It does not seem possible to solve for $\lambda_s(s)$ (or the fee $\phi(s, \omega) = \exp(u_m(s, \omega))$) analytically. Nor indeed is it even clear that a solution which satisfies this first-order condition would be solution to even the first-order problem because the manager's objective is not concave in general and therefore the first-order condition may not be sufficient. To see this, note that for a fixed reporting strategy (for example, when the correct signal is known in the second-best), the double integral in the manager's objective function (13) is affine (linear plus a constant), so that convexity of the cost function implies that the overall objective is concave. However, in the third-best the manager can vary the reporting strategy. In this case, the maximum across reporting strategies of the double integral is the maximum of affine functions and is therefore convex. The curvature in the cost function may or may not overcome the curvature in the optimized double integral. If not, the objective function fails to be concave and the first-order conditions may fail to characterize the incentive-compatibility constraint.¹² For example, in the limiting case of a proportional cost function, the objective is convex in effort (once we have optimized over reporting strategy), and the manager will never choose an interior effort level. In this case, any binding incentive-compatibility constraint will compare full effort with no effort, and will not be the same as the local condition.

When the first-order approach does work in the third-best, we interpret the final term in (28) to be the additional incentive needed at the margin to induce truthful

¹²Matters are somewhat more subtle than it might seem from the text, since the utility levels in the double integral are endogenous to the investor's choice problem. Therefore, we do not know how to specify a priori a level of convexity in the cost function $c(\cdot)$ large enough to ensure the manager's objective function is concave, since concavity still depends on the investor's specification of utility payoffs.

reporting. In a numerical example developed in the next section, this means that we give the manager a larger budget share as a reward for reporting a more extreme signal, to remove a possible incentive to be a "closet indexer" that mimics the index and expends low effort but collects fees appropriate for active management.

III Numerical Results

Now we turn to numerical results that compare the first-best, second-best, and third-best. For the "informed" and "uninformed" joint density of ω and s, we assume joint normality with the same marginals with (informed density) and without (uninformed density) correlation $\rho > 0$. We think of this as a model of market timing, with ω representing the demeaned log market return in the usual lognormal model over one year. Let $n(\cdot; \cdot, \cdot)$ be the normal density parametrized by mean and variance. Then $f^s(s) = n(s; 0, \sigma^2)$ is the density of s in either case, $f^{\omega}(\omega) = n(\omega; 0, \sigma^2)$ is both the unconditional density and the conditional density of the market state ω in the uninformed case, and $f^I(\omega|s) = n(\omega, \rho s, \sigma^2(1 - \rho^2))$ is the conditional density of the market state ω given s in the informed case. State prices are consistent with Black-Scholes and can be computed as the discount factor times the risk-neutral probabilities as $p(\omega) = e^{-r}n(\omega;\mu - r,\sigma^2)$. In these expressions, r is the riskfree rate, μ is the mean return on the market, and σ is the standard deviation of the market return. Without loss of generality, the signal s has mean 0 and the same variance as the log of the market return.

To facilitate the comparison of the cases, we vary the cost function to make the same effort level optimal in the first-, second-, and third-best. This removes the obvious distinction among the contracts that higher equilibrium effort implies a more informative signal and therefore more aggressive portfolios for both agents. By fixing ε exogenously, we isolate the differences among the contracts due solely to the addition of the IC constraints.

We work with discretized versions of $f^{I}(\omega|s)$, $f^{U}(\omega)$, and $p(\omega)$ with N market states and M signal states. In order to circumvent the difficulty imposed by the presence of $\lambda'_{s}(s)$ in this first order condition of the third-best problem we work with a discrete version in which the reporting constraint is replaced by two sets of reporting constraints. The first set imposes the restriction that reporting the state just higher than the true state is not optimal and the other does the same for reporting the state just lower than the true state. Together this makes 2(M-1) constraints. As the discretization becomes very fine this problem approximates the continuous state case.



Figure 1: Manager's Utilities: First-best Contract

The manager's utilities from the first best problem are plotted in Figure 1. The parameters used are $\mu = 0.15$, $\sigma = 0.2$, $\rho = 0.5$, r = 0.05, and $w_0 = 100$. We chose a cost of effort function such that $u_0 - c(\varepsilon) = .955$ and $c'(\varepsilon) = 0.2$ at $\varepsilon = 0.5$.

A visual inspection of the solution to the second and third-best contracts at these parameter values is not very instructive. However we can gain insight by examining the incremental changes in the contract when we move from first-best to second-best to third-best. Figure 2 plots the manager's utilities in the second-best minus the manager's utilities in the first-best. When signal and market are both high (or both low), $f^{I}(\omega|s) > f^{\omega}(\omega)$, so the manager is rewarded in those states. In the other corners of the distribution, the manager has less utility than in the first-best case. This provides the incentive to exert effort.

Figure 3 plots the manager's utilities in the third-best minus the manager's utilities in the second-best. The difference between these two contracts is that the



Figure 2: Manager's Utilities: Second-best minus First-best Levels

third-best provides incentives to truthfully report. A manager facing a secondbest contract would tend to be overly conservative in reporting to reduce the extra risk exposure the second-best contract offers compared to the first best. From the figure, we can see that compared to the second-best a manager reporting an extreme signal has a higher payoff and less risk.

The intuition for this is straightforward. In order to induce effort the manager is overexposed to the risk of the signal as in the bonus of the second-best contract. However a manager who may misreport will tend to try and report a signal that is too conservative in order to try and reduce this risk exposure. This may be related to plan sponsors' common concern that managers might be "closet indexers" who mimic the index but collect fees more appropriate for active managers.

Another way to understand the difference between the third- and second-best contracts is to look at the differences in payoff which are plotted in figure 4. Notice that in terms of dollars the difference between the two contracts is very subtle in low signal states of the world. When the signal is low the manager's pay is also low in the second-best contract and so marginal utility is high. This means that only a small increase in pay is required to induce the manager to report the correct state. However when the signal state is high the manager's pay is also high and so a very large bonus is required to induce truthful reporting. Thus the way to truthful reporting can be thought of as more of a "carrot" approach than a "stick".



Figure 3: Manager's Utilities: Third-best minus Second-best Levels

Conceptually, the third-best model seems more compelling than the first-best and second-best models because effort is probably not contractible and the manager's beliefs after exerting effort are not publicly verifiable. Nonetheless, explicit contracts observed in practice seem to look like our first-best and second-best solutions: fees based on a proportion of assets under management (as in the first-best), with or without additional compensation based on performance relative a benchmark (as in the second-best), are common, while explicit contracts that compensate directly for taking more extreme positions (as in our third-best solution) are not. There are several possible explanations for this apparent inconsistency between theory and practice. All of these explanations are outside the scope of our model, and we do not have strong views about which explanation is most accurate.

One possible explanation is that the first-best and second-best problems are better representations of the underlying economic problem, perhaps because there is a mechanism outside the model for handling the problem of truthful reporting or closet indexing. For example, perhaps site visits to the manager and examination of the records assure the investor that the manager is investing as intended. Or, it could be a reputation effect: we do observe monitoring for closet indexing in the hiring of managers, and this monitoring may produce a reputation-based incentive for taking the requisite risk. In general, there seems to be no reason to expect that alternative mechanisms will generate the correct incentives.



Figure 4: Manager's Payoff: Third-best minus Second-best Levels

A second possible explanation is that there is some psychological or organizational reason that people do not behave as in the third-best. For example, perhaps investors and managers do not realize there is an incentive problem or different people in a fund management firm choose effort level and the portfolio. Or, managers may like the feeling of honestly mapping the information into the signal while promises of effort are necessarily vague and underexertion of effort may be easier to rationalize. These explanations are not very useful as theory, since having this kind of explanation does not seem to put any restriction on behavior.

A third possible explanation is that there are reasons outside the model why managers want to take on too much risk. For example, convexity in the implicit reward from future fund flows or other business may overwhelm incentives in direct compensation to take too little risk. Or, it may be that managers who have an inflated view of their own abilities are the ones who tend to self-select to be in this business. Whatever the detailed reason, this explanation is consistent with the observed portfolio constraints, most of which seem to limit, not encourage, risk-taking. For example, investment guidelines may restrict industry rating or tracking error to keep the portfolio from deviating too much from a benchmark.¹³ This explanation seems generally consistent with empirical evidence of the influence of current returns on future business for mutual funds (as in Huang, Wei, and Yan (2003)).

A fourth possible explanation (not inconsistent with the others) is that the incentive to misreport may be small. In our numerical results, several choices of parameter values consistent with the first-order approach generated third-order solutions that were close numerically to the second-best, especially for parameter values for which the manager's information is not very informative about the market state (consistent with priors that the market is not too far from being efficient).

At this point, we do not have a favorite explanation (or even a comprehensive list of explanations) for the apparent inconsistency between our theoretical sensibilities (which suggest the third-best is the "correct" analysis) and practice (which looks a lot like our first-best and second-best solutions). We hope that future models or empirical work improve our understanding of this issue.

IV Conclusion

We have proposed a new model of optimal contracting in the agency problem in delegated portfolio management. We have shown that in a first-best world with log utility the optimal contract is a proportional sharing rule over the portfolio payoff. In a second-best world the optimal contract (if it exists) is a proportional sharing rule plus a bonus proportional to the excess return over a benchmark to give incentives to the manager to work hard. In a third-best world, such excess return strategies will provide incentives to work but will tend to make the manager overly conservative. These results have been demonstrated in the context of a realistic return model and the derived performance measurement criterion looks more like the simple benchmark comparisons used by practitioners than more elaborate measures such as the Jensen measure, Sharpe measure, or various marginal-utility

¹³Another justifi cation of this sort of restriction is enforcement of diversifi cation across diverse managers of a fund. If all managers, whether labeled large-cap growth or small-cap value, were to invest in similar mid-cap value portfolios, diversifi cation would suffer.

weighted measures. In addition, the optimal contract includes restrictions on the set of permitted strategies. These institutional features are more similar to practice than other existing agency models in finance.

We have only just started to tap the potential of this framework to tell us about agency problems in portfolio management. Although some of the general results extend to stock selection models as well as the market timing examples given in this paper, it would be interesting to see the exact form of the contracts for stock-pickers. Analyzing career concerns would be an interesting variant: in this case, the current client has to take as given the manager's incentives to demonstrate superior performance this period in order to attract new clients or achieve a larger wage next period. In this case, there is probably a limit to the extent to which the client can neutralize the impact of career concerns. It would also be interesting to consider problems in which the manager's utility function (as well as consumption) is bounded below, given that the actual economy has restrictions on indentured servitude. Rajan and Srivastava (2000) considers a simple model of delegated portfolio management with limited punishment. It would be interesting to see what limited punishment or career concerns would imply in our model.

In the model, we have obtained a lot of mileage from the transparent and frictionless markets assumption that allows us to look at an equivalent formulation in which the manager simply reports information and does not actually manage the money. However, there are aspects of performance (such as quality of execution) that are not handled adequately in this way. While institutions receive complete reports of which trades were made (and mutual fund performance reports can depend in this information in any necessary way as computed credibly by the custodian or consultant), even the full trade record combined with full quote and trade histories of each stock would not necessarily tell us what trading opportunities were available at each point in time. It would be useful to have a fuller exploration of when the reporting formulation is equivalent and of what happens otherwise. Another extension would include explicitly the two levels of portfolio management we see in practice, with the separation of responsibilities for asset allocation across asset classes and management of sub-portfolios in each asset class. The ultimate beneficiaries have to create incentives for the overall manager to hire and compensate the asset class managers, and this could be modeled as a hierarchy of agency contracts.

References

- Admati, A. R., and P. Pfleiderer, 1997, "Does it All Add Up? Benchmarks and the Compensation of Active Portfolio Managers," *Journal of Business*, 70, 323– 350.
- Almazan, A., K. C. Brown, M. Carlson, and D. A. Chapman, 2001, "Why Constrain Your Mutual Fund Manager?," Working paper, University of Texas at Austin.
- Basak, S., A. Pavlova, and A. Shapiro, 2003, "Offsetting the Incentives: Risk Shifting and Benefits of Benchmarking in Money Management," Working paper 4304-03, MIT Sloan School of Management.
- Bhattacharya, S., and P. Pfleiderer, 1985, "Delegated Portfolio Management," *Journal of Economic Theory*, 36, 1–25.
- Cox, J. C., and C. Huang, 1991, "A Variational Problem Arising in Financial Economics," *Journal of Mathematical Economics*, 20, 465–487.
- Demski, J. S., and D. E. M. Sappington, 1987, "Delegated Expertise," *Journal of Accounting Research*, 25, 68–89.
- Dybvig, P. H., L. C. G. Rogers, and K. Back, 1999, "Portfolio Turnpikes," *The Review of Financial Studies*, 12, 165–195.
- Faynzilberg, P. S., and P. Kumar, 1997, "Optimal Contracting of Separable Production Technologies," *Games and Economic Behavior*, 21, 15–39.
- Faynzilberg, P. S., and P. Kumar, 2000, "On the generalized principal-agent problem: decomposition and existence results," *Review of Economic Design*, 5, 23–58.
- Garcia, D., 2001, "Optimal Menus of Linear Contracts in Portfolio Selection Models," Working paper, Tuck School of Business Administration.
- Gómez, J.-P., and T. Sharma, 2001, "Portfolio Delegation under Short-selling Constraints," Working paper, Norwegian School of Management.
- Grossman, S., and O. Hart, 1983, "An Analysis of the Principal-Agent Problem," *Econometrica*, 51, 7–45.

- Hart, O., and B. Holmström, 1987, "The Theory of Contracts," in Truman F. Bewley (ed.), Advances in Economic Theory – Fifth World Congress. chap. 3, pp. 71–155, Cambridge University Press.
- Holmström, B., 1979, "Moral Hazard and Observability," *Bell Journal of Economics*, 10, 74–91.
- Holmström, B., 1984, "A Note on the First-Order Approach in Principal Agent Models," mimeo.
- Huang, J., K. D. Wei, and H. Yan, 2003, "Volatility of Performance and Mutual Fund Flows," Working paper, University of Texas at Austin.
- Kihlstrom, R. E., 1988, "Optimal Contracts for Security Analysts and Portfolio Managers," *Studies in Banking and Finance*, 5, 291–325.
- Laffont, J.-J., and D. Martimort, 2002, *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press, Princeton, NJ.
- Mirrlees, J., 1974, "Notes on Welfare Economics, Information, and Uncertainty," in Balch, McFadden, and Wu (ed.), *Essays on Economic Behavior Under Uncertainty*, North Holland, Amsterdam.
- Mirrlees, J., 1976, "The Optimal Structure of Incentives and Authority Within an Organization," *Bell Journal of Economics*, 7, 105–131.
- Myerson, R., 1979, "Incentive Compatibility and the Bargaining Problem," *Econometrica*, 47, 61–73.
- Ou-Yang, H., 2003, "Optimal Contracts in a Continuous-Time Delegated Portfolio Management Problem," *The Review of Financial Studies*, 16, 173–208.
- Palomino, F., and A. Prat, 2003, "Risk Taking and Optimal Contracts for Money Managers," *RAND Journal of Economics*, 34, 113–137.
- Rajan, U., and S. Srivastava, 2000, "Portfolio Delegation with Limited Liability," Working paper, GSIA, Carnegie Mellon University.
- Rogerson, W., 1985, "The First-Order Approach to Principal-Agent Problems," *Econometrica*, 53, 1357–1367.

- Ross, S. A., 1973, "The Economic Theory of Agency: The Principal's Problem," *American Economic Review*, 63, 134–139.
- Ross, S. A., 1974, "On the Economic Theory of Agency and the Principal of Preference Similarity," in Balch, McFadden, and Wu (ed.), *Essays on Economic Behavior Under Uncertainty*, North Holland, Amsterdam.
- Ross, S. A., 1979, "Equilibrium and Agency Inadmissable Agents in the Public Agency Problem," *American Economic Review*, 69, 308–312.
- Stole, L. A., 1993, "Lectures on the Theory of Contracts and Organizations, Chapters 1-2," Monograph in preparation. Available at http://gsblas.uchicago.edu/teaching/papers/lectures.pdf.
- Stoughton, N. M., 1993, "Moral Hazard and the Portfolio Management Problem," Journal of Finance, 48, 2009–2028.
- Sung, J., 1991, "Managerial compensation and corporate management under moral hazard," Ph.D. thesis, Washington University in Saint Louis.
- Sung, J., 1995, "Linearity with Project Selection and Controllable Diffusion Rate in Continuous-Time Principal-Agent Problems," *Rand Journal of Economics*, 26, 720–43.
- Sung, J., 2003, "Optimal Contracts under Adverse Selection and Moral Hazard: a Continuous-Time Approach," Working paper, University of Illinois at Chicago.
- Zender, J. F., 1988, "Asymmetric Information in Financial Markets," Ph.D. thesis, Yale.