Using Asset Allocation to Protect Spending

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The management of an educational endowment or other income-producing portfolio involves two strategic decisions at the fund level: asset allocation and choice of a spending rule. Traditionally, these two decisions are linked, for example, to preserve capital on average, but the optimal link would be more dynamic. This article describes a new protective strategy that links spending and asset allocation in a way that preserves spending power in down markets but participates significantly in up markets. The strategy is similar to constant proportions portfolio insurance, in that part of the fund is maintained in safe assets to preserve the value needed for continued expenditures. Like portfolio insurance, the strategy outperforms traditional strategies when markets are persistently up or persistently down but underperforms when portfolios are whipsawed by alternating ups and downs.

Managing portfolios used for current income involves two strategic decisions at the fund level: asset allocation and choice of a spending rule. Asset allocation is the decision of how much of the endowment to place in various broad asset classes, such as domestic equities, foreign equities, and government bonds. Choice of a spending rule is the decision of how much should be budgeted for removal from the endowment for current spending and how much should be left in to provide for future spending. Asset allocation and choice of a spending rule must be linked. This article proposes a protective strategy for spending and asset allocation that protects capital better than current practice but still participates significantly in up markets. For institutional context, we emphasize the application to educational endowments, but the policy is applicable to many different types of nontaxable accounts.

Improving Current Practice

Conceptually, endowment management involves an alternating sequence of spending decisions and asset allocation decisions. At the beginning of a year (or other planning period), the endowment officers look at the current size of the endowment and decide how much to extract for expenditure in the coming period and how much to leave in the endowment to invest for future spending. Then, they take what remains in the endowment and invest it in some mix of asset classes. Over the year, the portfolio may rise or fall in value, depending on whether fortune smiled on the investments and depending on any additional additions to or withdrawals from the portfolio. At the end of the year, the budget decision is made once again, followed by another asset allocation decision. Of course, actual decisions are planned in advance more than in this description and budgeted money may stay in the endowment until used, but in any case, choices alternate between spending and asset allocation.

In recent decades, universities have become increasingly sophisticated in linking budgets and investment policy. The two decisions are necessarily linked because the amount of funds available to one is given by the amount left over from or provided by the other. Current practice tends to link budget proportions to average long-term returns in order to maintain principal on average. Having more of a link between the two was once taboo because it was associated with an archaic (in managing university endowments but sadly not in trust management) limit on spending more than dividends plus interest (see Keane 1996 and Murray 1996). This link is static, however, and does not provide for dynamic feedback between the two. There is also a serious problem with the usual calculation of whether capital is preserved, but that is the topic for another paper.¹

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Figure 1 reports backtesting for 1946 through 1996 of the performance of a strategy that is a first approximation to current practice. For this strategy, half of the portfolio is in large-capitalization U.S. stocks and half is in three-month T-bills, with a spending rate of 4.5 percent per year. (Details of the backtests are given in Appendix A.) Panel A reports the size of the endowment (the inflation-adjusted portfolio value) and the spending amount, both assuming an initial investment of $100 million. The spending curve is the smoother because the amount changes only once a year. Panel B shows the portfolio composition corresponding to the strategy. To keep the proportions constant, funds are moved from the stock portfolio to the bond portfolio when the market goes up and from the bond to the stock portfolio when the market goes down.

A slightly more complex strategy is a better representative of current practice. In this strategy, asset allocation is dictated (within a band) by an asset-pricing model (such as the capital asset pricing model or arbitrage pricing theory) that quantifies the trade-off between risk and return. In this strategy, spending is also smoothed by being proportional to a moving average (say, over three or five years) of endowment values or unit values instead of the current value. The overall level of expenditure is set using a rule of thumb intended to preserve principal, on average. Figure 2 shows the effect of smoothing on spending.

Figure 2 is based on the same spending rule shown in Figure 1 except with annual spending of 4.5 percent of the average of the 60 most recent months’ real portfolio values. Averaging reduces the typical magnitude of year-to-year changes in spending, but it does not change the overall shape of expenditures over time. Indeed, in principle, if a more persistent decline in the stock market should occur than has occurred in this post-World War II sample, a much larger dip could occur under averaging than under the traditional rule. The reason is that expenditures following a persistent decline could be much greater than the 4.5 percent rate, which would result in significant erosion of capital.

The smoothed spending rule may reasonably serve the needs of an institution in many market environments, but it is an ad hoc rule with little basis in theory. To protect spending in a way that smoothing cannot, funds need to link asset allocation and the spending rule with an eye toward the underlying expenditures the endowment wants to protect. I specifically wish to consider strategies that can ensure that spending will never decline or, alternatively, strategies that ensure that spending will never fall faster than a prespecified rate of decline. Figure 3 provides backtests of the proposed strategy in which spending cannot decline.

In Figure 3, annual spending is increased as necessary (and only as necessary) to keep it from falling below 1.5 percent of the portfolio value, and spending is never decreased. For the portfolio choice, there is a protected part that is invested 100 percent in short-term Treasuries, and the remainder is invested 100 percent in stocks. The protected part is taken to be the annual spending rate divided by 3.5 percent.

Unlike current practice, the proposed strategy avoids large spending cuts while still allowing the fund to participate significantly in rising markets. For example, the large dip in real spending experienced by funds using the traditional strategy in the mid-1980s would not have occurred under the proposed strategy. The reason is that the proposed strategy would have protected a fund’s ability to maintain spending by moving out of risky assets and into safer investments as the market fell. In general, the strategy avoids spending cuts while still allowing significant participation in rising markets.

An important feature of the proposed strategy is that it requires spending rates that are less than what is now common. This reduction is necessary because maintaining for certain a perpetual spending level that is higher than the current endowment times the riskless rate is not possible. A modification of the strategy (described later) would permit a higher initial spending rate, but the cost is a process of declining rates. Figure 4 shows a backtest of this diluted version of the new type of strategy.

In Figure 4, annual spending is increased as necessary (and only as necessary) to keep it from falling below 4.5 percent of the portfolio value, and spending is never decreased. For the portfolio choice, there is a protected part that is invested 100 percent in short-term Treasuries, and the remainder is invested 133.33 percent in stocks and short 33.33 percent in Treasuries. The protected part is taken to be the annual spending rate divided by 8.5 percent. (Again, refer to Appendix A for more details about the backtesting.) Figure 4 shows that there is no free lunch: The higher the initial spending rate, the faster the spending rate must be able to decline and the more the strategy mimics the traditional strategy without protection. The cases explored in Figures 3 and 4 are extremes. In between, the higher the initial spending level, the larger the necessary maximum decline.

Protected Expenditure

Central to the proposed approach to coordinating asset allocation and the spending rule is the notion...
Figure 1. Performance and Portfolio Composition of a Simple Portfolio Strategy and Spending Rule, 1946–96

A. Performance

B. Composition
Figure 2. Performance and Portfolio Composition of a Portfolio Strategy and Spending Rule with Smoothing, 1946–96

A. Performance

B. Composition
Figure 3. Performance and Portfolio Composition of the Proposed Expenditure Protected Policy, 1946–96

A. Performance

B. Composition
Figure 4. Performance and Portfolio Composition of a Diluted Version of the Proposed Expenditure Protected Policy, 1946–96

A. Performance

B. Composition
of protected (or committed) expenditure. Under the proposed plan, at any point in time, the fund will have a minimal amount of future spending it can obtain from the portfolio no matter how badly the stock market performs. This is possible because as the value of the portfolio falls, the fund does not transfer more and more assets into risky investments as it would do in a fixed-proportions strategy. Instead, the fund keeps enough money in safe investments to deliver enough cash flow to cover the future protected expenditure. That is, the proposed portfolio is partitioned into a protected part and a risky part. The protected part contains immunized riskless investments whose payoffs are designed to produce the planned expenditure over time; the risky part is poised to capture the average expected returns above the riskless rate that represent the compensation for taking on risk.

"Protected expenditure" could refer to actual commitments the fund has made (for example, for salary or for funding a program), or it could represent the fund officers’ best judgment about the income pattern below which unacceptably large damage would be done. As some fixed date approaches, the protected expenditure at that date will usually rise, as increases in the portfolio value justify a commitment to more and more expenditure at that date. The full commitment is normally made at the time the allocation moves from the arena of investments into the budgeting process.

Of course, it would be nice to have protected expenditure at a high level for all future dates, but the fund is subject to the constraint that the present value of the future protected expenditures must be no greater than the value of the fund. If the portfolio is worth $100 million and the interest rate is 5 percent, the highest constant commitment to spending the fund can make in perpetuity is $5 million a year, and if the fund makes that commitment, it will be locked out of the stock market and the higher average returns available there. Moreover, if the fund wants the protected expenditure to grow over time, an even lower level of initial expenditure is available. For many institutions, spending should be protected in real (inflation-protected) terms so that, at worst, the purchasing power stays constant. In this case, the fund can probably treat short-term T-bills as a real riskless asset and expect from them a real return of, say, 3 percent. (The new issues of indexed T-bonds should allow managers to lock in spending power with even more confidence than in the past.) Nonetheless, T-bills have historically yielded only a slight premium above inflation, and it is unclear how high a real return a fund can plan on. Despite the fact that stocks tend to outperform Treasuries in the long term, the fund cannot use its investment in stocks to spend in perpetuity a larger amount than $5 million. The reason is that, although there is some probability that the fund can make a larger payout, there is also a significant probability of a shortfall. To assert otherwise is to state that the fund is certain that stocks will go up and that going long in stocks and short in the riskless asset is, in effect, a riskless arbitrage. Such cheerful optimism may be an appealing personality trait, but it is not a healthy attitude for an investment manager. It is exactly the perspective some people had a month before jumping out of the window in the crash of 1929. Should the fund worry about something that happens so infrequently? Of course, it should, at least in the case of university endowments, because the managers are in the business of managing the endowment of an organization that is supposed to survive for a long time. Fifty years is not a long enough time to experience all that can happen in a financial market, and the fund should be prepared to deal with whatever reasonable contingencies may occur over the endowment’s life.

Fund managers should resist drawing strong conclusions from the historical performance of the U.S. stock market for another reason—namely, that it has performed beyond expectations. Stock markets that were equally promising (or more promising) 100 years ago have performed less well or have disappeared entirely. Researchers study the U.S. stock market precisely because it is atypical in its survival and prosperity. For example, when Jorion and Goetzmann (forthcoming) considered equity returns from a variety of countries from 1921 through 1996, they found the average U.S. real rate of return in their sample to be 4.3 percent, whereas the average rate for all countries was only 0.8 percent. In the light of these statistics, it is difficult to make the case that current university endowment management can be relied on to preserve capital (even ignoring the additional concerns raised in Note 1).

A fund can establish the spending commitment in a number of ways. The commitment can be for a fixed dollar amount each year in perpetuity. Probably more reasonably, the commitment can be adjusted to maintain spending power in the face of inflation—increasing a dollar amount each year to offset inflation. Or the commitment can be set to decline over time, consistent with the view that maintenance of spending power is not so important and decreasing expenditure is not so devastating if it is gradual. Or the commitment may extend for a limited number of years in the future with a further extension each year only if that is prudent given the news about the portfolio.
The Proposed Strategy

In Dybvig (1995), the strategy proposed here was derived as the solution to an optimization problem (given in Appendix B). That analysis started with a description of the economic situation, which was then formalized in a choice problem and subsequently solved. Having the strategy that solves a reasonable optimization problem attests to the reasonableness of the strategy.

The proposed strategy is as follows: At a point in time \( t \), wealth (or size of the endowment) is \( w_t \), of which \( \alpha_t \) is invested in the risky portfolio (say, domestic equities or a favored mixture of various risky assets) and \( w_t - \alpha_t \) is invested in the riskless portfolios (short-term Treasuries or, better yet, inflation-protected Treasuries). Spending is at a rate of \( s_t \) dollars per year. The constant \( r \) is the rate of interest (presumably, a rate in excess of inflation). The part of the endowment placed in the risky portfolio is

\[
\alpha_t = K \left( w_t - \frac{s_t}{r} \right),
\]

and the amount placed in riskless assets is the remainder,

\[
w_t - \alpha_t = \frac{s_t}{r} + (1 - K) \left( w_t - \frac{s_t}{r} \right),
\]

where \( s_t/r \) is the wealth required to maintain current spending forever because it is the amount the fund would have to invest at the riskless rate to generate interest of \( s_t \) each year. Therefore, the portfolio has the cost of maintaining current spending forever, \( s_t/r \), invested in the riskless portfolio, and the remainder, \( w_t - (s_t/r) \), invested in fixed proportions of \( K \) in the risky portfolio and \( 1 - K \) in the riskless portfolio.

The spending rule is normally to keep spending constant in the short run. Spending is increased only as is necessary to maintain

\[
s_t \geq r^* w_t, \text{ with } r^* < r + d,
\]

where \( d \) is the permitted rate of decline. That is, spending can never fall below a critical proportion of wealth, \( r^* \). In the portfolio of Figure 3, \( K \) is set equal to 1, so the entire cushion is invested in the risky asset (\( r^* \) is set equal to 1.5 percent a year). Even taking \( K > 1 \) (more than 100 percent of the cushion in equities) need not ever lever the portfolio as a whole so long as \( K \leq r/(r - r^*) \).

The constants \( K \) (the risky proportion for the discretionary part of the portfolio) and \( r^* \) (the minimal spending rate) are parameters of the strategy that are chosen at the outset. My preference is to choose the parameters by experimenting with simulated returns to obtain the most appealing range of results in a number of simulated draws. An alternative is to base the parameters on the theoretical model (the problem discussed in Appendix B) that gives formal justification of the strategy. Unfortunately, the theoretical model relies on preference parameters that must be supplied.

As a constraint of the problem, spending is never decreased from the previous year. If fortune has smiled on the fund and the portfolio has increased in value over the year, then spending is increased only to the extent needed to maintain a spending rate of \( r^* \). Over time, spending can vary between a minimum of this critical rate \( r^* \) and a maximum of \( r \), depending on luck and skill in the risky investments. If \( s_t/w_t \) equals \( r \), then the portfolio is fully invested in bonds. When \( s_t/w_t \) is less than \( r^* \), spending is immediately raised to a new level, \( s_t^* \), where \( s_t^*/w_t = r^* \). The value is now the new level of spending that can never decrease.

The investment policy is tied to the spending rule. The rule works as if two separate accounts exist, although implementation of the strategy does not require separate accounts. One account, the “committed amount,” is nondiscretionary and is used for funding the current spending rate continued into the indefinite future. The other account, the “cushion,” is discretionary and is invested in fixed proportions of stocks and bonds. The spending rule keeps expenditures at a fixed level until the fund needs to raise the level to maintain a minimum spending rate. Therefore, when the portfolio value goes down, the fund maintains spending, but when the portfolio value goes up enough, the fund increases spending. As the portfolio value goes down, a smaller and smaller proportion of the portfolio is at risk in the discretionary part, which permits the fund to maintain enough value in the committed amount of the portfolio to keep from ever having to reduce spending. The strategy’s qualitative features are summarized in Exhibit 1.

Recall that Figure 3 shows the (inflation-adjusted) performance of this strategy during the same post-war period examined in Figures 1 and 2. A comparison of the figures shows that the spending under the proposed policy is qualitatively quite different from the spending in current practices: It never falls. Also, the portfolio choice is quite different: The dollar amount left in riskless assets is much more stable, especially in response to falling equity markets. The late 1970s and early 1980s were a difficult time for many endowments, and Figure 2 shows one reason: In the traditional strategy, a falling market implies falling real spending. In contrast, in the proposed strategy, spending is preserved. Important distinctions between the traditional strategy and the proposed strategy are summarized in Exhibit 2.
Implementation

Before implementing the strategy, practitioners should take the following essential steps:

- Plan to follow the strategy for a period of years. In general, switching destroys value, even if the strategy is reasonable both before and after the switch (see Dybvig 1988).
- Choose the parameters of the strategy. In the simplest form of the strategy, the parameters are the critical minimal spending rate and the risky proportion for the discretionary part of the portfolio. The two approaches to choosing the parameters are described in this section.
- Develop a plan for transition to the new strategy. Spending under the proposed strategy is likely to be less than under current rules. Therefore, phasing in the new program is advisable. An example would be to move a fixed proportion each year into the new program.

This section first considers how to choose the parameters and then considers other implementation issues.

Choosing Parameters. The first approach to choosing $K$ and $r^*$ uses simulations of portfolio value to clarify the trade-offs involved. I recommend this approach. For one reason, simulating investment results from the strategy helps a user to internalize the implications of the strategy. In the second approach, which takes the model as a literal description of what the strategy entails, the parameter choice is based on the user’s preferences. The advantage of using simulation to evaluate parameter choices is that it does not require the user to buy into the theoretical model or determine the preference parameters in the model. Theoretical models—particularly models of preferences—are necessarily abstractions of reality. The advantage of the second approach is theoretical precision.

Parameter values from simulation. I suggest generating plots of the annual spending from the endowment and its value based on the endowment’s current practice and the proposed method. The plots would be similar to those shown in Figures 1–4 for historical returns, but simulated portfolio returns should be used in addition to historical returns. The simulation will give a more accurate picture of the range of reasonable performance scenarios than will looking at the single scenario in the historical record. I recommend starting with the same stochastic model of portfolio returns as used in the formal decision problem in Appendix B, although additional realistic properties of returns (such as stochastic volatility or time-varying risk premiums) could, of course, be included. I have placed a Java applet that simulates the strategy at http://dybfin.wustl.edu/java/ratchet.

Parameter values from underlying preferences. In this approach, the preference parameters to be specified are the pure rate of time discount, $\delta$, and the degree of relative risk aversion, $R$, that enter the objective function in the formal decision problem in Appendix B. The idea is to think about your preferences for different random and nonrandom outcomes, and to compute from these preferences what your values of $\delta$ and $R$ must be. For example, you can figure out $R$ by thinking about what increase for sure (3 percent? 4 percent?) would be considered just as good as a 50/50 chance of increasing by 10 percent or getting no increase at all. Or, if you were
originally planning 10 percent more spending next year than this, how much more spending would you require initially if you are only to get a 5 percent increase next year. These sorts of questions allow you to quantify your preferences for risk and for spending at different points in time. Choosing parameter values that make the preferences give answers that match your answers to these questions is one scientific way of deciding the appropriate parameters for your strategy. In principle, this ensures that the optimization problem will give the solution to a choice problem that is a very good approximation to the one you should be solving.

Having made the best case for this approach, I am still skeptical of the quality of the choices coming from it. Most people do not find these parameters intuitively satisfying, and if the assumptions about preferences are not accurate or if these examples are not very representative of true preferences about actual results, the parameters may not be useful. The problem is that the sample questions used to elicit preferences are not similar to realistic problems in endowment management. Furthermore, if you are not careful in selecting parameters, the choice problem will not have any solution and the formulas will not be defined. For example, if δ and R are too small, any plan will be dominated by delaying spending to have even larger spending later. In this case, if you try to use the formulas in Appendix B, you will find you are taking the square root of a negative number and the asset allocation or spending rule will be undefined.

**Committed Spending.** The theoretical model takes the committed amount to be maintenance of current spending (probably in real terms) forever. A simple alternative (considered in Dybvig 1995) is to allow the committed amount to decline at no more than a given rate. If the largest permissible decline is zero, the result is the model described here. Setting a decline for the committed amount would make it possible for the endowment fund to support a higher level of current spending. In the basic strategy with no decline, the fund can spend, at most, the current endowment value times the real interest rate; when consumption is allowed to decline, the fund can spend the current endowment value times the real interest rate plus the rate of decline in the committed amount. Of course, nothing comes for free: The greater the allowable rate of decline, the weaker the protection against lean times. As the permitted decline increases, the strategy looks more and more like the traditional strategy.

The revised strategy with the committed amount declining at a rate \(d\) is given by revising Equations 1–3 as follows: The part of the endowment to be placed in risky assets is

\[
\alpha_t = K \frac{w_t - s_t}{r + d},
\]

and the amount placed in riskless assets is

\[
w_t - \alpha_t = \frac{s_t}{r + d} + (1 - K) \left(\frac{w_t - s_t}{r + d}\right),
\]

where \(d\) is the permitted rate of decline.

Spending normally declines by a proportion \(d\) per unit of time and is increased only as necessary to maintain

\[
s_t \geq r^* w_t, \text{ with } r^* < r + d.
\]

The portfolio strategy is different from the strategy with no decline (given by Equations 1–3) because the protected part of the endowment is smaller. The investment required to maintain committed spending forever is less because in this strategy, the endowment can spend not only the riskless return but also from a proportion of the original investment corresponding to the decrease in commitment. The consumption strategy is different because consumption declines when the endowment is not at the minimum \(r^*\) proportion. The strategy is also different in a more subtle way because the minimum spending rate may be larger than the risk-free rate (so long as it is less than \(r + d\)).

In general, the committed spending could be any shape a manager might choose. For example, you might view your endowment as having a five-year horizon and your committed amount as extending into Year 5. Next year, you would make a commitment for what now looks like Year 6, and that commitment could be larger or smaller than the commitment for Year 5. You might also adjust the commitments in Years 1–5. Of course, if you reduce the commitments for Years 1–5, you are making a mockery of the process because the commitment then means little or nothing. In any of these cases, the committed part of the portfolio would include bonds that replicate the cash flows you are committed to.

**Uncertain Interest Rates.** Given a fixed interest rate, figuring how much must be protected to honor the endowment’s future commitment is easy. If the endowment commits to $500,000 a year forever and the interest rate is 5 percent, then the fund requires $10 million in the riskless asset to honor the commitment because the annual interest on $10 million is 5 percent of $10 million, or $500,000. For more-complicated cash flow streams (for example, commitments of various amounts in the next five years), one can use the well-known concept of present value to compute how much must be invested to meet the endowment’s annual commitments.
In practice, of course, no one knows what the interest rate will be in the future. Therefore, the fund cannot simply invest the money in short-term investments and be assured of later having the value needed. If the fund is trying to maintain nominal (dollar) spending levels, a whole literature is available on immunization of portfolio value against interest rate shocks to ensure fixed payments. The simple approach is to match cash flows, although this approach is limited to maturities no greater than the longest available bond. More-sophisticated approaches are based on matching single or multiple duration measures or, better yet, on a sensible multifactor model of the term structure of interest rates.

Universities are probably more interested in protecting the spending power of commitments than in maintaining nominal spending levels, which means immunizing in real terms. The good thing about protecting real spending power is that real interest rates seem to be much less volatile than nominal rates, which reduces the reinvestment risk. There is a longstanding question of what investment can be considered riskless or nearly riskless in real terms, and one reasonable answer is that rolling over short-term Treasuries is pretty nearly riskless in real terms, especially over longer horizons. Now there is a good answer, namely, that the new inflation-linked T-bonds are essentially riskless in real terms (at least insofar as the price index underlying the bonds is appropriate for a university’s expenses). These bonds are available for only a few maturities at present, which means that matching cash flows is impractical for now. However, the low volatility of real rates means that reinvestment risk is minimal for these instruments and not many maturities will probably be needed for a good hedge. It will be interesting to see just how liquid this market is and how many maturities will become available.

It is interesting to note that there may be ways to convert real commitments to nominal commitments. An example is the purchase of a building with borrowed money, which trades the real need for offices and classrooms for a nominal commitment to make mortgage payments. A related issue is the hedging power of buying the building; it is a more perfect hedge for the spending need than would be investment in indexed Treasuries. Although the mortgage introduces inflation risk (because the payments are nominal), that risk can be hedged by guaranteeing nominal (rather than real) contributions of the endowment to the budget. In general, a university could consider hedging its demands for energy, salaries, and other expenses separately, with different contracts for each (although the ability to hedge salaries accurately is probably limited).

Illiquid Assets. A potential advantage that universities have over short-term investors is that they need less liquidity and can profit from long-term investments. Illiquid assets do not pose a particular problem in the discretionary part of the portfolio, especially if most of the portfolio is liquid and available for any movement called for by the strategy to the committed part. In the committed part, illiquid assets may be appropriate if they give the required return pattern with little or no risk. Such a pattern may be uncommon among illiquid assets, however, because illiquidity is usually the result of information asymmetry about a risky asset.

Coordination with Other Sources of Income. Some universities have endowments large enough to represent the majority of the institution’s budget, but many do not. So, one question about a protected strategy is how useful is a guarantee on part of invested funds that are themselves a small part of the budget. The answer probably depends on the use of the proceeds from the endowment and on the reliability of other sources of income. If the proceeds are earmarked for a specific purpose that represents a commitment (e.g., to fund a faculty chair), then treating the spending as committed spending and using the techniques described here may make sense. If the stability of the other sources of funds is questionable, having this guaranteed amount in the endowment in case of a shortfall elsewhere may be prudent. This generic motive for protecting the endowment is not a consideration if the endowment is truly small. For example, if spending from the endowment goes into a general budget in which it represents 1 percent of spending that moves around significantly from year to year, nothing done with the endowment will have a big impact (unless important expenses cannot be funded out of the usual budget because of, say, a legislative restriction on how a state school can use tuition and funds from the state).

In thinking about committed spending, new money from operations or from donations may make it reasonable to permit spending at a rate higher than the real interest rate and still expect to maintain real spending. Relying on donations to keep growing at the same rate is potentially dangerous, however, because distant future donations are not certain.

Transitional Issues. One interesting question is what is the optimal strategy for making a
transition from a traditional program to a protected program of the sort described here. The most difficult part of the transition is that to maintain a real guarantee, the fund may need to set a lower initial spending rate than was inherited from the old program. The transition can be made in a number of reasonable ways. One way is to switch only a portion of the money at one time. Another, if the fund is fortunate enough to have significant new donations, is to put only the new monies into the protected portion. One good time to switch is when spending rates are lower than usual—for example, as they often would be following a bull market.

**Conclusion**

The approach to endowment management described here links the asset allocation decision and the spending rule through the concept of protecting assets to fund committed expenditures. Traditionally, these two decisions have not been linked in this way. In the proposed protective strategy, which is similar to the strategy of constant proportions portfolio insurance, part of the fund is kept in safe assets to preserve the value needed for continued expenditures. The strategy outperforms traditional strategies when markets are persistently up or persistently down but underperforms when portfolios are whipsawed by repeated up and down moves.

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**Notes**

1. Unfortunately, preservation of capital on average (which is the basis of the usual calculation) does not necessarily preserve capital in any meaningful sense. For example, betting the entire endowment on a horse race with fair odds preserves capital on average, but it does not conform to any reasonable notion of preservation of capital. If the mean return of the portfolio is 10 percent with a standard deviation of 30 percent, expected inflation is 3 percent, and spending is 4.5 percent, the usual calculation would show that capital is preserved because spending is less than the mean return net of expected inflation. However, the real value of a portfolio with these characteristics tends almost surely to zero over time because the value at a date far in the future is similar to the payoff in a lottery ticket. Steve Heston and I are preparing a paper on this topic.

2. The idea of survivorship bias in measuring U.S. market returns was first introduced to me by Pete Kyle in the early 1980s.

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**References**


Appendix A. Details of the Backtesting

The backtesting was based on the monthly stocks, bonds, bills, and inflation data provided by Ibbottson Associates and distributed by CRSP. Each simulation was based on two asset classes—stocks and bonds. For the stocks, we used the S&P 500 Index. U.S. Treasury securities with an average duration of three months were used for the bond returns. All simulations used real returns, so all values are in terms of purchasing power. Real (inflation-adjusted or constant-dollar) returns were computed from the U.S. Consumer Price Index (CPI).\(^{A1}\)

Real returns were computed as follows: Let \( y \) be the ordinary return on an asset in a month, which is defined to be the change in dollar value over the month (inclusive of any coupons, dividends, splits, rights, etc.) divided by the dollar value at the start of the month. Let \( i \) be the inflation rate over the month, the change in the CPI over the month divided by the value at the start of the month. Then, the real return on the asset over the month is given by

\[
Y = \frac{1 + y}{1 + i} - 1,
\]

which is the same as “the change in purchasing power” divided by “the purchasing power at the start of the month.” The quantities \( y, i, \) and \( Y \) are often expressed as percentages and may be annualized by multiplying by 12.

Testing the new type of strategy with protected real spending requires knowledge of investment returns that are riskless in real terms. Indexed Treasury bonds were not available during the sample period, so the short-term Treasury return was used as a proxy for the real riskless asset; over time, the returns to this asset tend to move up and down with inflation. Computing the strategy also required evaluation of future real cash flows. For this purpose, I assumed a fixed real rate of 3.5 percent, which is higher than the average real return on short-term Treasuries but is consistent with current market rates on the new indexed Treasuries. Monthly data were used for the back tests, and the strategy was updated monthly.

\(^{A1}\) Concerns have been raised about what is the right price index to use for what purpose and also about potential biases in the CPI. For this study, however, I considered adjusting for inflation to be better than no adjustment and believe the results are reasonable.

Appendix B. Details of the Formal Model

The formal choice problem and the derivation of its solution are given in Dybvig (1995); this appendix contains a brief introduction. The exact meaning of all the symbols is probably unclear to those without formal training in probability theory and continuous-time finance, but anyone can get an intuitive feel for what is going on without that background. The formal choice problem is the same as the famous problem studied by Merton (1973) except for the additional constraint that spending can never fall.

The formal decision problem is as follows: Choose the spending rule \( s_t/r \) and risky portfolio investment \( \alpha_t \) to maximize the objective function

\[
\mathbb{E} \left[ \int_0^\infty U(s_t) \exp(-\delta t) \, dt \right]
\]

subject to

\[
(\forall t > 0) s_t \geq s_t \text{ (nondecreasing spending)},
\]

and

\[
(\forall t) w_t \geq 0 \text{ (nonnegative wealth)},
\]

where \( w_t \) solves the budget equation:

\[
dw = w_t r dt + \alpha_t (\mu dt + \sigma dZ_t - rd dt) - s_t dt,
\]

subject to the initial-wealth constraint, \( w_0 = W_0 \). The utility function is the power function,

\[
U(s_t) = \frac{s_t^{1-R}}{1-R}.
\]

The variable \( t \) indexes time, and both choice variables can depend on time and on past portfolio returns. The objective function says that we want to maximize the sum (\( \int \) indicates integral, the continuous analog of a sum) over time of instantaneous benefits \( U(s_t) = s_t^{1-R}/(1-R) \), weighing benefits at \( t \) by a weight \( \exp(-\delta t) \) that is declining over time. The parameter \( \delta \) is the pure rate of time discount; the larger its value, the less importance is being placed on later spending.

The nondecreasing spending constraint ensures that spending can never decline. The nonnegative wealth constraint ensures that the fund cannot borrow forever without repayment and also rules out doubling strategies.
Finally, the budget equation describes how wealth changes. If wealth were invested solely in the riskless investments without any spending, the fund would receive the riskless rate \( r \) per unit of time. For the amount \( \alpha \) invested in risky assets, the fund receives a mean return \( \mu \) per unit time and random return \( \alpha dZ \), and loses out on the riskless return \( r \) per unit of time. Subtracted from the sum is the spending \( s \), per unit of time.

The solution to this choice problem is given by Equations 1–3. The constant \( K \) in the solution to the formal decision problem is

\[
K = \frac{\mu - r}{R^*\sigma^2},
\]

where

\[
R^* = \frac{\sqrt{\left(\delta + \kappa - r\right)^2 + 4r\kappa - \left(\delta + \kappa - r\right)}}{2r}
\]

is a number between 0 and 1 and

\[
\kappa = \frac{\left(\mu - r\right)^2}{2\sigma^2}.
\]

The constant \( r^* \) in the solution to the problem is

\[
r^* = \frac{R - R^*}{R}.
\]

There are many more details and also extensions, including the case of limited decline, in Dybvig (1988).

Having specified the form of the problem and its solution, we are prepared to return to the question of how to think about the parameters of the utility function if we are to obtain “parameters from underlying preferences.”

The pure rate of time discount \( \delta \) enters the objective function directly, and risk aversion enters through the “felicity” function, \( U \). You can get a handle on these parameters by asking such questions as: What riskless percentage increase in spending would be just as attractive as a 50/50 chance of no increase or a 10 percent increase? The answer to this question would be comparable to the theoretical \( x \) value when solving the equation

\[
U[(1 + x)s] = 0.5U(s) + 0.5U(1.1s),
\]

which for the \( U \) functions being considered here does not depend on \( w \) but does depend on \( R \). Once \( R \) is known, you can determine \( \delta \) by asking: What percentage increase in spending both this year and next would be just as attractive as a 10 percent increase next year only? The answer to this question would be comparable to the theoretical \( x \) value found in solving

\[
U[(1 + x)s] + \exp(-\delta)U[(1 + x)s] = U(s) + \exp(-\delta)U(1.1s).
\]

As an example, suppose this year’s budget is $10 million and a 50/50 chance of no increase or an increase to $11 million is just as attractive to you as having a budget of $10.3 million for sure. You know that

\[
U(s) = s^{1-R}/(1 - R),
\]

which is the felicity function that indicates how you feel about various spending levels. Therefore, relative risk aversion can be computed by solving

\[
\frac{10.3^{1-R}}{1 - R} = 0.5\frac{10^{1-R}}{1 - R} + 0.5\frac{11^{1-R}}{1 - R}
\]

for \( R \), which can be done by trial and error or by plotting the difference between the two sides as a function of \( R \). For this example, \( R \) is approximately 0.498.

Now suppose that you are indifferent between flat spending this year and spending next year at $10.45 million and spending $10 million this year and $11 million next year; that is, you are willing to make do with a little less if you can do some of the spending now. Then, you can find \( \delta \) as the solution to

\[
\frac{10.45^{1-R}}{1 - R} + \exp(-\delta)\left(\frac{10.45^{1-R}}{1 - R}\right) = \frac{10^{1-R}}{1 - R} + \exp(-\delta)\left(\frac{11^{1-R}}{1 - R}\right),
\]

where \( R = 0.498 \) as already computed. Solving this equation (which you can do algebraically or numerically), you find \( \delta \) is approximately 0.177.

The optimal strategy is then given by Equations 1–3, where \( r^* \) and \( K \) are given by formulas in this appendix. These formulas use the values of \( R \) and \( \delta \) computed here and the values for the return parameters—the real risk-free rate, \( r \); the mean return of the risky assets, \( \mu \); and the standard deviation of the risky assets’ return, \( \sigma \).